Disclosure and Financial Markets: Introductory Course

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ACCT611

Syllabus: outside work

- **Readings prior to every class**
  - Participation is an essential component of the class; come prepared with questions
  - Two 15 minute presentations per student.
  - I will assign dates to make it easy, though the dates can be swapped.

- **Periodic problem sets**
  - Complete in Mathematica and/or LaTeX

- **Final exam**
  - Covers math behind the core models
  - Serves as preparation for the prelim

Syllabus: overview of the course

- First half of the class focuses on related models: exogenous information releases and their impacts on market outcomes
  - "Competitive" trading models, Grossman and Stiglitz (1980), Hellwig (1980)
  - "Strategic" trading models, Kyle (1985)

- Second half of the class will discuss several frameworks in which information releases are influenced by a strategic agent.

Preamble: why theory?

Economic theory is sometimes critiqued.

- Some view it as unnecessary.
- Some view the assumptions as too strong for meaningful inferences.

First things first: we don't *always* need a model.

- In many cases, verbal intuition is sufficient to generate valid hypotheses.
- Or, maybe we care only about prediction.

Preamble: why theory?

In other cases, models may be very useful.

- Verbal arguments may be slippery.
  - The assumptions underlying an argument may be hard to parse out.
  - Thus, we could spend years disagreeing without figuring out the essence of our disagreement.
- In some cases, we may not have good data or the right data to answer a specific question (or large measurement error).
- Models can enable you to uncover intuition that you wouldn't have otherwise thought of.

What about unrealistic assumptions?

- Any good model must make simplifying, unrealistic assumptions.
- Ideally, the assumptions abstract from:
  - quantitatively unimportant forces, or
  - important forces that are unrelated to the force under consideration.

Solow's viewpoint

Robert Solow (1956): "All theory depends on assumptions which are not quite true. That is what makes it theory. The art of successful theorizing is to make the inevitable simplifying assumptions in such a way that the final results are not very sensitive. A 'crucial' assumption is one on which the conclusions do depend sensitively, and it is important that crucial assumptions be reasonably realistic. When the results of a theory seem to flow specifically from a special crucial assumption, then if the assumption is dubious, the results are suspect."

Friedman's viewpoint

Milton Friedman (1953): "Truly important and significant hypotheses will be found to have 'assumptions' that are wildly inaccurate descriptive representations of reality, and, in general, the more significant the theory, the more unrealistic the assumptions (in this sense)."

Preamble: reading theory – some tips

- Reading and working with models benefits greatly from knowing when to focus on big picture versus when to invest in understanding the details.
- Following every detail might be helpful at first but can become burdensome. Often better to start by thinking at a high level and then finding the important details.
- Don't become discouraged: Models that seem hard to start might seem obvious after working with them for a while.

Today's agenda

1. **Math background**
2. Taxonomy of disclosure theory
3. Lambert, Leuz, and Verrecchia (2007)
4. Estimation risk
5. Asymmetric information
6. Mathematica exercise

Basic math tools for disclosure theory

A classic "single stock" set up:

- There is a continuum of price-taking investors trading in a stock with payoffs $\tilde{v}$ that are normally distributed.
  - Slightly different than Bertomeu and Cheynel, who have finite # of investors, but will be convenient when studying private information.
- Investor i perceives that $\tilde{v} \sim N(\mu, \sigma^2)$.
- Investors have CARA utility with risk aversion γ:

$$u(W) = -\exp(-\gamma W).$$

  Let their initial wealth be W_0.
- The stock has a supply of Q per capita.
- Investors can trade in a bond that pays off 1 and has price fixed at 1 $\Leftrightarrow$ costless borrowing and lending

Sometimes referred to as a "CARA/normal" model.

Basic math tools for disclosure theory

The classic result: Investor i's demand, D_i, satisfies:

$$D_i = \frac{\overbrace{\mu - P}^{mean\ -\ price}}{\underbrace{\gamma\sigma^2}_{R.A.\ *\ variance}}$$

The firm's equilibrium price satisfies:

$$P = \underbrace{\mu}_{mean} - \underbrace{\gamma Q \sigma^2}_{R.A.\ *\ supply\ *\ variance}.$$

This expression holds even when μ and σ^2 result from various information signals (will become important when we introduce disclosure).

Basic math tools for disclosure theory

We will use the following result in the proof.

Remark

Suppose $\tilde{\mathbf{v}} \sim MVN(\mathbf{m}, \mathbf{V})$ *(multivariate normal). Then,*

$$-E\left[\exp\left(-\mathbf{t}'\tilde{\mathbf{v}}\right)\right] = -\exp\left(-\mathbf{t}'\mathbf{m} + \frac{1}{2}\mathbf{t}'\mathbf{V}\mathbf{t}\right).$$

- This is the "moment generating function" (MGF) of a multivariate normal.
- Expected utility calculations with CARA utility $u(W) = -\exp(-\gamma W)$ take this form; terminal wealth involves an inner product of demands and asset payoffs.
- In the single asset case, we will use a simple special case of this result: if $\tilde{v} \sim N\left(\mu, \sigma^2\right)$, then:

$$-E\left[\exp\left(-t\tilde{v}\right)\right] = -\exp\left(-t\mu + \frac{1}{2}t^2\sigma^2\right).$$

Proof.

Because there are many small investors, they take prices as a given ("perfect competition"). Each investor solves:

$$\begin{aligned} & \arg\max_D -E\left[\exp\left(-\gamma W_0 - \gamma D\left(\tilde{v} - P\right)\right)\right] \\ = \quad & \arg\max_D -\exp\left(-\gamma W_0\right) E\left[\exp\left(-\gamma D\left(\tilde{v} - P\right)\right)\right] \\ = \quad & \arg\max_D -E\left[\exp\left(-\gamma D\left(\tilde{v} - P\right)\right)\right]. \end{aligned}$$

Notice we have "wealth independence." Now, from the formula earlier, this equals:

$$\arg\max_D -\exp\left(-\gamma D\left(\mu - P\right) + \frac{1}{2}\gamma^2\sigma^2 D^2\right).$$

Proof (continued).

Applying monotonic transformations,

$$\begin{aligned} & \arg\max_D -\exp\left(-\gamma D\left(\mu - P\right) + \frac{1}{2}\gamma^2\sigma^2 D^2\right) \\ = & \arg\max_D \underbrace{D\left(\mu - P\right) - \frac{1}{2}\gamma\sigma^2 D^2}_{\text{certainty equivalent}}. \end{aligned}$$

Differentiating and solving:

$$D = \frac{\mu - P}{\gamma\sigma^2}.$$

We now use market clearing to solve for prices in equilibrium.

$$\begin{aligned} \text{Demand} &= \text{Supply} \\ \Leftrightarrow \int D_i di &= \int Q \, di \\ \Leftrightarrow \int \frac{\mu - P}{\gamma\sigma^2} di &= Q \\ \Leftrightarrow P &= \mu - \gamma\sigma^2 Q. \end{aligned}$$

Multi-asset case

- Now, suppose instead that there are N assets and investors perceive them as having a multi-variate normal distribution with common mean parameter $\mathbf{m}$ and variance matrix $\mathbf{V}$.
- Suppose each has per-capita supply Q and let $\mathbf{Q}$ be an $N \times 1$ vector with each entry equal to Q.

Proposition

The investors' equilibrium demands satisfy:

$$\mathbf{D_i} = \frac{1}{\gamma}\mathbf{V}^{-1}(\mathbf{m} - \mathbf{P}).$$

The firm's equilibrium prices satisfy:

$$\mathbf{P} = \mathbf{m} - \gamma \mathbf{V}\mathbf{Q}$$

Proof.

- The derivation is almost identical. The MGF for a multivariate normal yields:

$$\begin{aligned} & \arg\max_{\mathbf{D}} - \exp\left(-\gamma \mathbf{D}'(\mathbf{m} - \mathbf{P}) + \frac{1}{2}\gamma^2 \mathbf{D}'\mathbf{V}\mathbf{D}\right) \\ = \; & \arg\max_{\mathbf{D}} \mathbf{D}'(\mathbf{m} - \mathbf{P}) - \frac{1}{2}\gamma \mathbf{D}'\mathbf{V}\mathbf{D} \end{aligned}$$

- Recall the matrix calculus rule for symmetric $\mathbf{V}$: $\frac{\partial \mathbf{D}'\mathbf{V}\mathbf{D}}{\partial \mathbf{D}} = 2\mathbf{V}\mathbf{D}$.
- So, the first-order condition is:

$$0 = \mathbf{m} - \mathbf{P} - \gamma \mathbf{V}\mathbf{D}$$

Using the equilibrium condition $\mathbf{D} = \mathbf{Q}$ yields $\mathbf{P} = \mathbf{m} - \gamma \mathbf{V}\mathbf{Q}$.

Two fundamental rules and their implications

1. Law of iterated expectations:

$$\underbrace{E\left\{E\left[\tilde{v}|\tilde{y}\right]\right\}}_{avg.\ posterior\ mean} = \underbrace{E\left[\tilde{v}\right]}_{prior\ mean}.$$

- Suppose $\tilde{v}$ is the firm's "fundamental value" and $\tilde{y}$ is a news release, e.g., earnings.
- This tells us that in a "risk-neutral" world, i.e., $P_t = E_t[\tilde{v}]$, prices cannot change on average around the release.
- Commonly used when doing calculations: expectations over multiple variables can be evaluated "one at a time:"

$$E\left[f\left(\tilde{v},\tilde{y}\right)\right] = E\left[E\left[f\left(\tilde{v},\tilde{y}\right)|\tilde{v}\right]\right].$$

Two fundamental rules and their implications

2. Law of total variance:

$$\underbrace{E\left\{Var\left[\tilde{v}|\tilde{y}\right]\right\}}_{avg.\ posterior\ uncertainty} = \underbrace{Var\left[\tilde{v}\right]}_{prior\ uncertainty} - \underbrace{Var\left\{E\left[\tilde{v}|\tilde{y}\right]\right\}}_{variation\ in\ beliefs}.$$

- Perhaps you've seen this in the case of R^2

  unexplained variance = total variance - explained variance
- **Implies:**
  1. On average, information always reduces uncertainty as captured by the variance (but a specific signal need not lower the variance).
  2. Ignoring the risk premium (i.e., $P = E[\tilde{v}|\tilde{y}]$): the more earnings move prices around, the more they reduce investor uncertainty.

Basic math tools for disclosure theory

Bayesian updating with normals comes up repeatedly. In the most general case, use Wikipedia:

and use Mathematica to invert the matrix Σ_{22}.

Basic math tools for disclosure theory

In simpler cases, this is not necessary. Suppose that:

$$\begin{aligned} \tilde{y} &= \tilde{v} + \tilde{\varepsilon} \\ \tilde{v} &\sim N\left(\mu, \sigma^2\right) \\ \tilde{\varepsilon} &\sim N\left(0, \sigma_\varepsilon^2\right) \\ & \quad \tilde{v} \text{ and } \tilde{\varepsilon} \text{ are independent.} \end{aligned}$$

Let $\eta = \sigma^{-2}$ and $\eta_\varepsilon = \sigma_\varepsilon^{-2}$, the *precisions* of $\tilde{v}$ and $\tilde{\varepsilon}$. Then, we have a few standard formulas:

$$\begin{aligned} E\left[\tilde{v}|\tilde{y}\right] &= \frac{\sigma^2\tilde{y} + \sigma_\varepsilon^2\mu}{\sigma^2 + \sigma_\varepsilon^2} = \frac{\eta\mu + \eta_\varepsilon\tilde{y}}{\eta + \eta_\varepsilon} = \mu + \frac{\sigma^2}{\sigma^2 + \sigma_\varepsilon^2}\left(\tilde{y} - \mu\right). \\ Var\left[\tilde{v}|\tilde{y}\right] &= \frac{\sigma^2\sigma_\varepsilon^2}{\sigma^2 + \sigma_\varepsilon^2} = \left(\eta + \eta_\varepsilon\right)^{-1}. \end{aligned}$$

Basic math tools for disclosure theory

$$
\begin{aligned}
E\left[\tilde{v}|\tilde{y}\right] &= \frac{\sigma^2\tilde{y}+\sigma_\varepsilon^2\mu}{\sigma^2+\sigma_\varepsilon^2} = \frac{\eta\mu+\eta_\varepsilon\tilde{y}}{\eta+\eta_\varepsilon} = \mu + \frac{\sigma^2}{\sigma^2+\sigma_\varepsilon^2}\left(\tilde{y}-\mu\right). \\
Var\left[\tilde{v}|\tilde{y}\right] &= \frac{\sigma^2\sigma_\varepsilon^2}{\sigma^2+\sigma_\varepsilon^2} = \left(\eta+\eta_\varepsilon\right)^{-1}.
\end{aligned}
$$

- Note the last expression tells us precisions are additive, which is why people sometimes use them.
- This formula generalizes readily to multiple signals.
- In many models, you can manipulate the signals to get them into the "truth-plus-noise" form.
- When in doubt, use the matrix approach.

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A taxonomy of disclosure theory

1. **Ex-ante commitment to release information**.

Applications: Mandatory disclosures, e.g., earnings announcements, 10-K's, 8-K's, proxy statements

Important questions:

- What are the effects of such information on:
  - Asset prices
  - Trade
  - Investor welfare
  - Firms' real decision making (e.g., investment decisions)
- How can one best design such policies (e.g., conservatism vs. aggressiveness) to affect desired outcomes?

A taxonomy of disclosure theory

2. **Costly manipulation of an information signal.**

Applications: earnings management, real and accrual.

Important questions:

- What determines the amount of earnings management?
- How does earnings management influence:
  - Credibility
  - Investment
  - Firm value
  - Investor uncertainty
  - Stock prices
- Can we reconcile specific patterns in observed earnings management activity (e.g., manipulation around cutoffs)?

A taxonomy of disclosure theory

3. **Discretion in the release of a verifiable disclosure.**

Applications: earnings forecasts, 8-K's, conference calls, MD&A, press releases. Broad.

Important questions:

- How frequently do managers disclose voluntarily?
- How does a lack of disclosure influence a firm's price/returns?
- How does the competitive environment affect disclosure decisions?
- How does voluntary disclosure interact with investment decisions?

May be the right counterfactual for evaluating disclosure mandates.

A taxonomy of disclosure theory

4. **Purely unverifiable disclosure.**

Applications: MD&A, social media, press releases, informal communication within the firm.

Important questions:

- What prevents agents from grossly lying, when doing so is seemingly costless?
- What determines how much an agent can communicate?
- What is the nature of communication and how does it influence agents' behavior?

Focus for the remainder of today

Today's focus: pure exchange models of information disclosure.
Typically operates in the "disclosure commitment" framework.

These models have four key exogenous features:

1. A cash flow distribution (parsimonious way to capture uncertainty over firm value). hence the term pure exchange
2. Investor utility functions.
3. Some market mechanism (e.g., how do demands map into prices, how do investors determine their demands).
4. Information signals: public and/or private.

Differs from the framework in "asset pricing 101," which often takes properties of *returns* rather than cash flows as the primitive construct.

Pure exchange models of disclosure

- **From here:** derive prices and other market outcomes of interest.

- **Some important questions we might want to answer:**
  1. How do stock prices and volume react to news?
  2. How does information quality impact liquidity, expected returns/cost of capital, price efficiency, etc.?
  3. How does accounting information interact with other forms of information, e.g., investors' private information?
  4. How does accounting information impact welfare?
  5. When/why might regulating disclosure be optimal?

Pure exchange models of disclosure

What do we abstract from?

1. The specifics of accounting.
   - We think about it purely as a Bayesian signal.
   - The details of how various accounting procedures map into this signal is not generally something we think about.
   - In some sense, this makes the models more general: we want to know how better information affects things, regardless of what drove the improvement.
2. Often, discretion in the accounting release.
3. Often, the interaction between accounting and investment decisions.

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Lambert, Leuz, and Verrecchia (2007)

Lambert, Leuz, and Verrecchia (2007) considers disclosure and prices in a multi-asset economy.

- This paper is very highly cited in the empirical literature.
- It is an instructive model for practicing the basic math tools.
- It further will illustrate some of the different forces that are relevant for thinking about disclosure's effects on returns in light of the forces of diversification.
- However, we will have to keep in mind in making sense of the results that they consider disclosure by only a single firm.
- We will see the results are typically misinterpreted in empirical work.

Basic multi-asset model of disclosure: set up

- Let's start with the prices we calculated for firms in an N-asset economy:

$$\mathbf{P} = \mathbf{m} - \gamma \mathbf{V}\mathbf{Q}.$$

  This yields the following price for firm i:

$$P_i = m_i - \gamma Cov\left(\tilde{v}_i, \sum_{j=1}^{N} \tilde{v}_j\right) Q.$$

- Now, suppose firm 1 releases a disclosure that equals its true cash flows plus independent noise:

$$\tilde{y}_1 = \tilde{v}_1 + \tilde{\varepsilon}_1$$

  where $\tilde{\varepsilon}_1 \sim N\left(0, \sigma_\varepsilon^2\right)$ is independent of all other variables. Then, we have:

$$P_i\left(\tilde{y}_1\right) = E\left(\tilde{v}_i | \tilde{y}_1\right) - \gamma Cov\left(\tilde{v}_i, \sum_{j=1}^{N} \tilde{v}_j \middle| \tilde{y}_1\right) Q.$$

  In general, there is a spillover.

Relation to CAPM

Let $\tilde{M} \equiv \sum_{j=1}^{N} Q\tilde{v}_j$ denote the "market" cash flows. Then, we can alternatively rewrite the price as:

$$\begin{aligned} P_i(\tilde{y}_1) &= E(\tilde{v}_i|\tilde{y}_1) - \gamma Cov\left(\tilde{v}_i, \tilde{M}|\tilde{y}_1\right) \\ &= \underbrace{E(\tilde{v}_i|\tilde{y}_1)}_{expected\ CF} - \beta_i(\tilde{y}_1) * \underbrace{\gamma Var\left(\tilde{M}|\tilde{y}_1\right)}_{market\ risk\ premium}, \end{aligned}$$

where $\beta_i(\tilde{y}_1) \equiv \frac{Cov\left(\tilde{v}_i, \tilde{M}|\tilde{y}_1\right)}{Var\left(\tilde{M}|\tilde{y}_1\right)}$.

- The risk premium equals the product of the firm's beta and the market risk premium.
- This looks like the standard CAPM, but is in terms of cash flows rather than returns, and includes a "disclosure."

Results

Proposition (Lambert, Leuz, Verrecchia Prop 2)

Following the disclosure, the covariance of firm 1's cash flows with any other firm's cash flows satisfies:

$$Cov\left(\tilde{v}_1, \tilde{v}_j | \tilde{y}_1\right) = Cov\left(\tilde{v}_1, \tilde{v}_j\right) \frac{Var\left(\tilde{\varepsilon}_1\right)}{Var\left(\tilde{\varepsilon}_1\right) + Var\left(\tilde{v}_1\right)}.$$

Furthermore, the same relation holds at the market level:

$$Cov\left(\tilde{v}_1, \tilde{M} | \tilde{y}_1\right) = Cov\left(\tilde{v}_1, \tilde{M}\right) \frac{Var\left(\tilde{\varepsilon}_1\right)}{Var\left(\tilde{\varepsilon}_1\right) + Var\left(\tilde{v}_1\right)}.$$

- Always lower than the respective prior covariances.
- No change when $Var\left(\tilde{\varepsilon}_1\right) \to \infty$.
- Equals zero when $Var\left(\tilde{\varepsilon}_1\right) \to 0$.
- Increases in the variance of $\tilde{\varepsilon}_1$.

Proof

Proof.

- We can calculate the conditional covariances via the "matrix inversion" approach for Bayesian updating.
- We have:

$$Var\begin{pmatrix} \tilde{v}_1 \\ \vdots \\ \tilde{v}_N \\ \tilde{y}_1 \end{pmatrix} = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1N} & \sigma_1^2 \\ \sigma_{12} & \ddots & & & \sigma_{12} \\ \vdots & & \ddots & & \vdots \\ \sigma_{1N} & \cdots & \cdots & \sigma_N^2 & \sigma_{1N} \\ \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1N} & \sigma_1^2 + \sigma_\varepsilon^2 \end{pmatrix}$$

Proof

Proof (continued).

Now,

$$Var\begin{pmatrix} \tilde{v}_1 \\ \vdots \\ \tilde{v}_N \end{pmatrix}\Bigg| \tilde{y}_1 \Bigg) = \begin{pmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1N} \\ \sigma_{12} & \ddots & & \\ \vdots & & \ddots & \\ \sigma_{1N} & \cdots & \cdots & \sigma_N^2 \end{pmatrix}$$

$$-\frac{1}{\sigma_1^2 + \sigma_\varepsilon^2}\begin{pmatrix} \sigma_1^2 \\ \sigma_{12} \\ \vdots \\ \sigma_{1N} \end{pmatrix}\begin{pmatrix} \sigma_1^2 & \sigma_{12} & \cdots & \sigma_{1N} \end{pmatrix}$$

The 1-j^{th} entry in this matrix is:

$$\sigma_{1j} - \frac{\sigma_1^2 \sigma_{1j}}{\sigma_1^2 + \sigma_\varepsilon^2} = \sigma_{1j}\frac{\sigma_\varepsilon^2}{\sigma_1^2 + \sigma_\varepsilon^2}.$$

□

Results

- Note this implies the expected price after the disclosure is higher:

$$\begin{aligned} E\left[P_1\left(\tilde{y}_1\right)\right] &= E\left[E\left(\tilde{v}_1|\tilde{y}_1\right) - \gamma Cov\left(\tilde{v}_1, \tilde{M}|\tilde{y}_1\right)\right] \\ &= m_1 - \gamma Cov\left(\tilde{v}_1, \tilde{M}|\tilde{y}_1\right) \\ &> m_1 - \gamma Cov\left(\tilde{v}_1, \tilde{M}\right). \end{aligned}$$

- This captures the conventional idea that disclosure reduces risk perceptions and thus lowers the risk premium.
- Roughly speaking, this implies there are positive expected returns on the disclosure date (why?).
- Define the post-disclosure cost of capital as $E\left(\tilde{r}_1|\tilde{y}_1\right) = E\left(\frac{\tilde{v}_1 - P_1}{P_1}|\tilde{y}_1\right)$. Then, we have:

$$Cost\ of\ Capital = \frac{1}{\frac{E\left(\tilde{v}_1|\tilde{y}_1\right)}{\gamma Cov\left(\tilde{v}_1, \tilde{M}|\tilde{y}_1\right)} - 1}.$$

Results

$$Cost\ of\ Capital = \frac{1}{\frac{E(\tilde{v}_1|\tilde{y}_1)}{\gamma Cov\left(\tilde{v}_1,\tilde{M}|\tilde{y}_1\right)} - 1}.$$

Disclosure's "direct" effect: reduction in $Cov\left(\tilde{v}_1, \tilde{M}|\tilde{y}_1\right)$.

- But, this effect applies to all firms, not just the disclosing firm. The above equation holds for all firms, not just firm 1.
- Moreover, the conditional covariance drops for all other firms as well! You can check this as an exercise using the matrix formula.
- Intuition: disclosure reduces both "systematic" and "idiosyncratic" risk. Idiosyncratic component is not priced; systematic component applies to all firms.

Results

$$Cost\ of\ Capital = \frac{1}{\frac{E(\tilde{v}_1|\tilde{y}_1)}{\gamma Cov\left(\tilde{v}_1, \tilde{M}|\tilde{y}_1\right)} - 1}.$$

Disclosure's "indirect" effect: positive signals raise $E\left(\tilde{v}_1|\tilde{y}_1\right)$, which lowers the firm's beta and the cost of capital.

- Applies most strongly to the disclosing firm.
- Intuition: fixing a firm's risk, if it performs better, it is *as if* it holds more cash, a zero-beta asset.
- *Caveat:* Somewhat dependent upon statistical structure. Doesn't arise in CRRA/log-normal models, which capture "continuous re-investment."
  More common to look at "dollar returns" in CARA/normal models to avoid this effect!

Results

Figure: Source: Bertomeu and Cheynel (2016)

Indirect effect and empirical work

Disclosure's "indirect" effect has a strong empirical prediction: accounting news that predicts greater future earnings should predict *lower* future betas and *lower* returns.

Contradictory evidence: Firm profitability *positively* predicts returns (e.g., Ball and Brown (1968); Novy-Marx (2013); Fama and French (2015)). Possible explanations:

1. Profitable firms are mispriced (in particular, undervalued).
2. Profitability is correlated with another source of firm risk that is more potent than the indirect effect.
   - Not as far fetched as it might sound. Remember these regressions control for valuation ratios, e.g., P:B.
   - For a given price, a firm that is perceived as risky *must* be more profitable. (profitability and risk "conditionally correlated" given price)

Neither effect enters the LLV framework (why?).

Direct effect and empirical work

- **Critical assumption underlying the direct effect:** Only one firm discloses.
  - The effect becomes trivially small when all firms disclose.
  - Intuition: a single firm's disclosure offers little incremental information on the market as a whole over and above all other information sources.
  - While it does provide incremental information on the firm's idiosyncratic value, this is not priced.
- **Indeed, empirically**: a company's earnings provide only a small amount of information on market-level performance, conditional on other firms' earnings, macro-indicators, etc., e.g., Bonsall, Fischer, and Bozanic (2013).
- **Thus:** the direct effect only applies to market-level changes in information quality (e.g., adoption of new reporting requirement).
  - Moreover, even market level info has no impact on the CoC when endogenizing the risk-free rate – see my notes "Rethinking Disclosure and the Cost of Capital"

Direct effect and empirical work

Yet empirical work often focuses on the broad-sample impact of firm-specific disclosures on the cost of capital by running regressions:

$$CoC_i = \alpha + \beta * DiscQual_i + \varepsilon_i$$

How can we break this stark result/*when* should we expect a relation between accounting information and the cost of capital?

1. Short-sale constraints + disclosure impacts investor disagreement (Miller (1977)).
2. Investment responds to accounting information (Gao (2012), Cheynel (2013)).
3. Disclosure resolves more/less uncertainty for good or bad news, and there is noise trade (Cianciaruso, Marinovic, and Smith (2021), Banerjee, Marinovic, and Smith (2021)).

But in these settings, information can increase cost of capital!
Related "puzzle:" there are positive expected returns on EA date ("earnings announcement premium").

Direct effect and empirical work

What about rational (or irrational) under-diversification?

- In theory, this might cause idiosyncratic risk to be priced, i.e., to increase the cost of capital. Thus, accounting info might reduce the cost of capital on average, as hoped.
- **But:** if anything, idiosyncratic risk seems to be priced in the wrong direction (Ang et al. (2006); though a debate rages over this)!

In sum: the relation between accounting info and the cost of capital is:

- A central question in empirical research;
- Has been the subject of decades of work;
- Has important policy implications.

Yet, it is not a resolved topic; theory and empirical work are disconnected. Moreover, the theory is often misunderstood and miscited.

Notes on supply and single-firm models

- Supply plays a key role in determining how information affects the risk premium in these models. Remember:

$$\mathbf{P} = \mathbf{m} - \gamma \mathbf{V}\mathbf{z}$$

  where $\mathbf{z}$ is the per-capita supply. If $\mathbf{z} = \mathbf{0}$, then there is no risk premium and no cost-of-capital effects.
- We typically want to think about a large economy where firms' idiosyncratic risks are diversifiable. This is incorporated into the model when we explicitly model many assets.
- But, many papers examine single-firm models.
  - In these models, if you include a non-zero risk premium ($z > 0$), you must think of this as arising from the firm's exposure to systematic risk.
  - To avoid focusing on the minuscule direct effect, if you want to analyze, say, firm-specific information, you have to set supply to zero.

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Estimation risk: a brief note

- LLV 2007 assumes parameters are known. Some prior work looks at uncertainty over model parameters. Often referred to as “estimation risk.”

- Ultimately, uncertainty over model parameters is not all that different than conventional uncertainty. Suppose that, for a given value of an unknown parameter $\tilde{\delta}$, the cash flow distribution is $f_{\tilde{v}|\tilde{\delta}}(\cdot|\cdot)$. Then, the ex-ante cash flow distribution is:

$$f_{\tilde{v}}(x) = \int f_{\tilde{v}|\tilde{\delta}}(x|t)\, f_{\tilde{\delta}}(t)\, dt.$$

  But, this is just another distribution, and we could always have started with this distribution. Sometimes, this changes the shape of the distribution

- Thus, estimation risk is only different if we assume investors treat it differently. Large “ambiguity aversion” literature assumes investors treat parametric uncertainty by assuming the worst or discounting it more heavily than standard uncertainty

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Asymmetric information

- We will next consider the case in which investors have different beliefs/different information.

- We will focus on a setting with only one stock for simplicity; in this case, our earlier price reduces to just:

$$P = E[\tilde{v}|\tilde{y}] - \gamma Q Var[\tilde{v}|\tilde{y}].$$

- Let's first look at how the previous results generalize to the case in which investors have exogenous heterogeneous beliefs, with different "precisions."
  - Suppose investor *i* believes that: $\tilde{v} \sim N(m_i, \tau_{v,i}^{-1})$.
  - Notice investors can differ in both their subjective means and subjective precisions.

Asymmetric information

Recall:

$$D_i = \frac{m_i - P}{\gamma \tau_{v,i}^{-1}}.$$

Applying market clearing, $\int D_i di = Q$, we get that the firm's price satisfies:

$$P = \frac{\int \tau_{v,i} m_i di}{\int \tau_{v,i} di} - \gamma Q \left[\int \tau_{v,i} di \right]^{-1}.$$

- The firm's price now reflects a "precision-weighted average" of investors' beliefs.
  - Intuitively: investors who are more confident trade more intensely on their beliefs.
- The risk premium reflects the average investor information precision.

Asymmetric information

$$P = \frac{\int \tau_{v,i} m_i di}{\int \tau_{v,i} di} - \gamma Q \left[\int \tau_{v,i} di \right]^{-1}.$$

- This expression holds even when m_i and $\tau_{v,i}$ result from various information signals.
- Because the risk premium is determined by the average investor's precision, information asymmetry per se does not influence this risk premium see Lambert, Leuz, and Verrecchia (2012)
  - Assumes optimistic investors are on average no more overconfident, i.e., $\tau_{v,i}$ and m_i are uncorrelated.
  - Always holds with normal distributions; Banerjee, Marinovic, and Smith (2021) finds it does not hold more generally.
- Disclosure has qualitatively similar effects as in the base model. Both direct and indirect effects on the cost of capital.
  - But, quantitative effects can differ because there are now other "competing" sources of information.

Asymmetric information

- Let's now formally move to the canonical model of trade with asymmetric information. That is, we will now consider differences in beliefs that endogenously result from investors observing different signals.
  - Hellwig (1980) is the classic reference; Grossman and Stiglitz (1980) include information acquisition.
  - In equilibrium, price partially, but not fully, reveals investors' private information.

- I will follow the notation in Goldstein and Yang's review.

Asymmetric information: set up

- **Consider a single-firm version of the model.**
  - Following our original model: continuum of price-taking CARA investors, stock payoffs $\tilde{v} \sim N\left(0, \tau_v^{-1}\right)$, per-capita stock supply Q, bond with return normalized to 1 in unlimited supply.

- All investors observe a public signal $\tilde{y} = \tilde{v} + \tilde{\eta}$ where $\tilde{\eta} \sim N\left(0, \tau_\eta^{-1}\right)$.

- **New:** Suppose now investors $i \in [0, \mu]$ are "informed," in that they observe private signals:

$$\tilde{s}_i = \tilde{v} + \tilde{\varepsilon}_i,$$

  where the errors $\tilde{\varepsilon}_i \sim N\left(0, \tau_\varepsilon^{-1}\right)$ are independent.

- The remaining investors $i \in [\mu, 1]$ are "uninformed" and do not observe private signals.

Asymmetric information: set up

- Furthermore, suppose there are *noise/liquidity* traders who demand $\tilde{x}$ units of the stock, where $\tilde{x} \sim N\left(0, \tau_x^{-1}\right)$ is independent of other variables in the model.

- Investor *i* can include the price as part of their information set.
  - Can think of this as submitting limit orders, but the economic force this captures is more general.
  - **Broadly:** captures the thought process when traders see a higher price that other traders might know something they don't.
  - If you see a stock you hold jump 15% but there was no public news, what do you think?
    - *Someone might know something I don't.*
    - *Or, maybe they think they do, but really don't ("noise traders").*
    - *Or, maybe a large shareholder received a big capital inflow and is sizing up their positions.*

Deriving the equilibrium

Conjecture an equilibrium in which price is linear in $\tilde{v}, \tilde{x}$, and $\tilde{y}$:

$$P = p_0 + p_y \tilde{y} + p_v \tilde{v} + p_x \tilde{x}.$$

Thus, $\frac{P - p_0 - p_y \tilde{y}}{p_v} = \tilde{v} + \frac{p_x}{p_v} \tilde{x}$.

- Let $\rho = \frac{p_v}{p_x}$, which we can think of as the informativeness of price.
- Then, the above signal is a truth-plus-noise signal with precision $\rho^2 \tau_x$.
- So, applying Bayes' rule for normals,

$$E\left[\tilde{v} | \tilde{s}_i, \tilde{y}, \tilde{P}\right] = \frac{\tau_\eta \tilde{y} + \tau_\varepsilon \tilde{s}_i + \rho^2 \tau_x \frac{P - p_0 - p_y \tilde{y}}{p_v}}{\tau_v + \tau_\eta + \tau_\varepsilon + \rho^2 \tau_x};$$

$$Var\left[\tilde{v} | \tilde{s}_i, \tilde{y}, \tilde{P}\right] = \frac{1}{\tau_v + \tau_\eta + \tau_\varepsilon + \rho^2 \tau_x}.$$

Breon-Drish (2015 ReStud) shows that the linear equilibrium is unique among the continuous class and, for a broad family of distributions, there is a "generalized linear equilibrium"

Deriving the equilibrium

Remaining steps:

1. Substitute investors' beliefs into their demand functions.
2. Substitute into the market clearing condition and solve for price as a function of p_0, p_y, p_v, and p_x.
3. Find the coefficients in this price function on y, v, and x. Solve the system of equations to find the equilibrium values of p_0, p_y, p_v, and p_x.

This yields:

$$p_0 = \frac{\gamma Q}{\mu\tau_\varepsilon + \tau_v + \tau_\eta + \rho^2\tau_x}; \quad p_x = \frac{\rho\tau_x + \gamma}{\mu\tau_\varepsilon + \tau_v + \tau_\eta + \rho^2\tau_x};$$

$$p_v = \frac{\mu\tau_\varepsilon + \rho^2\tau_x}{\mu\tau_\varepsilon + \tau_v + \tau_\eta + \rho^2\tau_x}; \quad p_y = \frac{\tau_\eta}{\mu\tau_\varepsilon + \tau_v + \tau_\eta + \rho^2\tau_x}$$

where $\rho = \frac{\mu\tau_\varepsilon}{\gamma}$.

Some features of equilibrium

- Price reacts to both informed trade and liquidity trade.
- Liquidity/noise traders affect prices both because:
  - Traders cannot distinguish their demands from informed demands ("adverse-selection" effect)

    This would go away if z were observable! High-frequency trade literature builds on this.
  - The risk-averse traders must take the other side of their positions, which exposes them to risk ("demand" or "inventory risk" effect)

    They buy/sell when liquidity traders sell/buy, and require a price decrease/increase.
- An aside: the "demand" effect should, in theory, grow small in a large economy (e.g., Petajisto 2009 JFQA)
  - Consider doubling the outstanding shares of a typical stock. Remains a tiny part of a diversified investor's portfolio $\Rightarrow$ the additional risk this presents is small
  - In practice, there does appear to be a demand effect (index inclusion effect, mutual fund fire sales)
  - Related to demand-based asset pricing (Koijen and Yogo 2019 JPE)

Disclosure and liquidity

- Liquidity in the model can loosely be thought of as how the price would differ if noise traders were to sell vs. buy a unit of stock.

- Suppose a liquidity trader comes to the market demanding 1 share. Then,

$$Ask = E[\tilde{P}|\tilde{x} = 1] = p_0 + p_x;$$
$$Bid = E[\tilde{P}|\tilde{x} = -1] = p_0 - p_x.$$

  So, the spread equals $2p_x$.

- Now, $2p_x = 2\frac{\rho\tau_x + \gamma}{\mu\tau_\varepsilon + \tau_v + \tau_\eta + \rho^2\tau_x}$, which decreases in τ_η. So, the spread narrows with disclosure quality.

- **Intuition:** better information means that the risk-averse investors are more willing to take the other side of noise traders' positions because of reduced risk (weakening the demand effect) *and* adverse selection.

Disclosure, price efficiency, and volatility

- We can think about **price efficiency** as the inverse of residual uncertainty over $\tilde{v}$ given the signal $\tilde{y}$ and $\tilde{P}$, $Var[\tilde{v}|\tilde{y}, \tilde{P}]^{-1}$. This can be shown to increase in disclosure quality τ_η.
  - Note this is slightly different than the risk premium/cost of capital, which captures the average investor's uncertainty.
  - This construct is closer to the information available to a naive observer who considers only price.

- We can think of **future return volatility** as $Var[\tilde{v} - \tilde{P}]$. This too can be shown to decrease in τ_η.
  - However, the variance of prices *today* rises in τ_η.

Information acquisition: Grossman and Stiglitz (1980)

To study information acquisition, let's first consider a simpler, classic paper: Grossman and Stiglitz (1980).

- Analyzes a setting similar to the one just studied, but where traders choose whether to learn a **common signal** s at a cost $c > 0$.

- Interior equilibrium where informed and uninformed investors coexist
  - Price only partially reveals the traders' signal, as in the setting just studied
  - Key insight: costly acquisition rules out a fully revealing price system
  - If prices fully revealed the signal, investors could free ride on price instead of paying to learn it directly

Information acquisition: Grossman and Stiglitz (1980)

Equilibrium characterized by *indifference* between acquiring and not acquiring info:

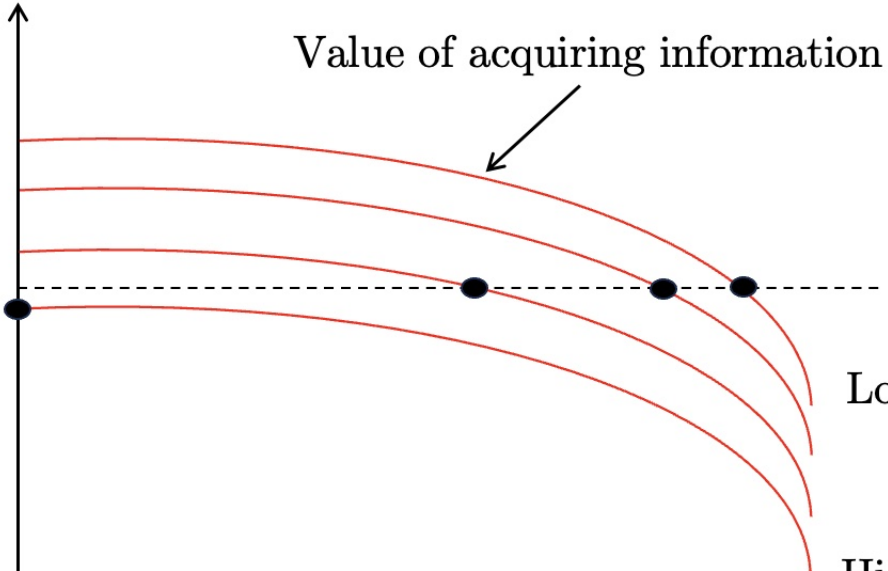
$$\frac{\text{InformedExpectedUtility}}{\text{UninformedExpectedUtility}} = 1.$$

Through miraculous simplifications covered in the appendix of Slides 4, this reduces to:

$$\frac{\overbrace{var[\tilde{v}|s]}^{\text{informed variance}}}{\underbrace{var[\tilde{v}|P]}_{\text{uninformed variance}}} = \overbrace{\exp(-2c)}^{\text{information cost}},$$

which has a convenient analytical solution.

Equilibrium with fixed-cost information acquisition



Equilibrium with fixed- & variable-cost info acquisition

Goldstein and Yang conduct a similar analysis, allowing both the decision to acquire information (fixed) and the decision of how much information to acquire (variable) to be costly. Moreover, remember traders acquire *different* signals.

Considerably more complicated:

- Traders either (i) acquire no info or (ii) acquire the optimal level of info τ_ε^*
- Equilibrium takes one of two forms; either (i) all traders acquire a signal and the benefits to doing so strictly exceed the costs, or (ii) traders are indifferent between acquiring a signal or not
- Equilibrium fraction of informed investors μ^* and optimal precision τ_ε^* satisfy a complex, two-dimensional system of equations
- No equilibrium in which perfectly precise private info is obtained by all investors, i.e., $\mu = 1$ and $\tau_\varepsilon = \infty$

Equilibrium with fixed- & variable-cost info acquisition

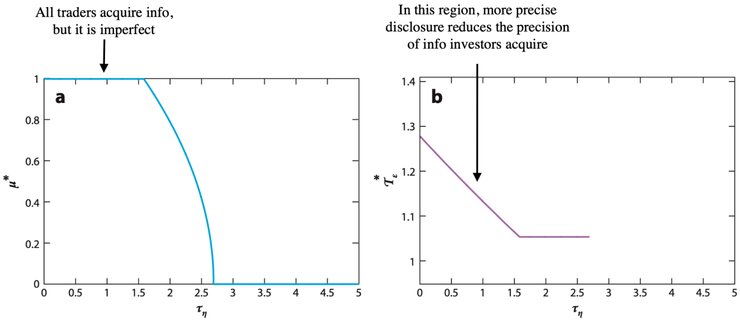


Figure: Equilibrium informed trade when cost to becoming informed is C_f as a function of disclosure quality τ_η. Source: Goldstein and Yang (2019).

Equilibrium with fixed- & variable-cost info acquisition

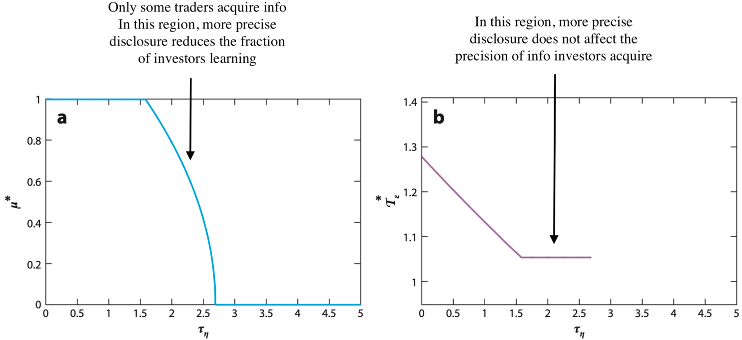


Figure: Equilibrium informed trade when cost to becoming informed is C_f as a function of disclosure quality τ_η. Source: Goldstein and Yang (2019).

Equilibrium constructs

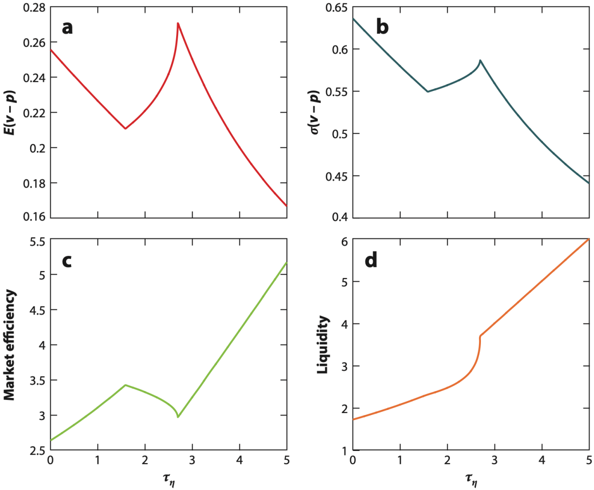


Figure: Crowding out effect can cause the positive effects of disclosure on market outcomes to reverse. Source: Goldstein and Yang (2019).

Recent applications

For those interested in recent applications:

- Kondor (ReStud): different trading horizons and the impact of public info on disagreement
- Nezafat and Schroder (RFS): endogenous short-sales constraints, learning from securities lending fees
- Glebkin, Gondhi, Kuong (RFS): funding constraints
- Goldstein, Kopytov, Shen, Xiang (Working): ESG information
- Goldstein and Yang (JF): two components of information relevant to firm value
- Banerjee, Breon-Drish, Smith (JFE): equity and debt, disagreement
- Smith (RFS): trade on information on both expected CF and risk in stock/derivative
- Farboodi et al. (RFS): valuing financial data (structural)

Miller (1977): Disagreement and short-sale constraints

Consider one final application, which leads to perhaps surprising results...

- Suppose there is a continuum of CARA investors with beliefs $\tilde{x} \sim N\left(\mu_i, \sigma^2\right)$, where $\mu_i \sim U\left[\mu - \Delta, \mu + \Delta\right]$.
- Per-capita supply of the asset is 1 and short sales are not allowed.

Investors' optimal demands are given by:

$$D_i = \begin{cases} \frac{\mu_i - P}{\gamma\sigma^2} & \text{if } \mu_i \geq P \\ 0 & \text{otherwise.} \end{cases}$$

- If this is not obvious, verify it using the Kuhn-Tucker condition.
- Assume $\Delta > \gamma\sigma^2$, which will ensure that, for some traders, short-sales constraints bind.

Miller (1977): Disagreement and short-sale constraints

Market clearing implies that aggregate demand is equal to 1:

$$\frac{1}{2\Delta}\int_{P}^{\mu+\Delta}\frac{\mu_i - P}{\gamma\sigma^2}d\mu_i = \frac{(\mu+\Delta-P)^2}{4\Delta\gamma\sigma^2} = 1.$$

This is a quadratic with two solutions. Only one solution satisfies $P < \mu + \Delta$ (which is clearly necessary for the market to clear):

$$P = \mu + \Delta - 2\sqrt{\gamma\sigma^2\Delta}$$

- If there were no short-sales constraints, the price would be $P = \mu - \gamma\sigma^2$ and so the difference in the prices is positive.
- Moreover, P in the constrained case increases in Δ.
- Exercise: how does disclosure change the expected price?

Agenda

1. Math background
2. Taxonomy of disclosure theory
3. Lambert, Leuz, and Verrecchia (2007)
4. Estimation risk
5. Asymmetric information
6. **Mathematica exercise**