Disclosure and Financial Markets Lecture 2

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Outline

Building on the competitive trading framework introduced in the previous lecture, this lecture considers three topics on disclosure and stock prices:

Outline

Attention and Disclosure Processing

Accounting regulations often determine how information is presented or aggregated rather than whether it is disclosed at all.

- Hence, issues of information processing and limited attention are fundamental
- Yet, models discussed so far assume that investors can costlessly & perfectly process all public information
- Considerable empirical literature on disclosure processing covered in empirical seminars and surveyed by Blankespoor, deHaan, and Marinovic (2020, JAE)

Evidence on Disclosure Processing

Recall: impact of inattention often measured using delayed reaction to news ("drift").

If information continues to predict returns after its release, it suggests

- Investors did not fully react to the news immediately
- The news subsequently diffuses among investors or is released in more easily processed forms, causing it to become more fully priced

Notable example is post-earnings announcement drift (PEAD), going back to Ball and Brown (1968).

Sample papers linking drift to limited processing:

- DellaVigna and Pollet (2009) - Friday earnings announcements have a smaller immediate and greater delayed reaction (PEAD)
- Hirshleifer, Lim, and Teoh (2009) - similar findings on days where there are more announcements
- Momentum literature, often attributed to limited attention to news (e.g., Da, Gurun, Warachka, 2014)
- Deeply connected to empirical work passive investing; passive investing is essentially continuous, complete inattention

Attention and Disclosure Processing Theory

Theoretical questions:

1. Pricing: How does costly processing / inattention impact size and speed of market reactions to information events?

2. Welfare: Who benefits and who loses when processing costs change?

3. Endogenous attention: What drives where investors place their attention and what are the consequences?

Attention and Disclosure Processing Theory

Recall Blankespoor, deHaan, and Marinovic breaks down processing costs into three categories:

1. "awareness cost" $\Leftrightarrow$ fixed cost of learning that info is available to be acquired
2. "acquisition cost" $\Leftrightarrow$ fixed cost of information acquisition
3. "integration cost" $\Leftrightarrow$ variable costs of information acquisition

We will start by focusing on awareness, specifically considering models where investors exogenously either (i) attend to and fully process a signal or (ii) are inattentive.

Will build on a key intuition from last class: Prices reflect a weighted average of active investors' expectations about firm cash flows.

Inattention: Rational Benchmark

Inattention and Disclosure: Rational Benchmark

Consider a variant of the CARA-normal setting studied in the last class.

Continuum of traders, CARA utility, per-capita asset supply $Q > 0$, risk aversion $\gamma > 0$.

1. Markets open, investors establish initial positions. *Not our focus!*
2. Information release $\tilde{y} = \tilde{v} + \tilde{\eta}$, $\tilde{\eta} \sim N\left(0, \tau_{\eta}^{-1}\right)$ is independent noise.
3. Markets open, price P_2.
4. Firm pays off $\tilde{v} \sim N\left(m, \tau_{v}^{-1}\right)$ to shareholders.

Suppose only a fraction $\lambda \in (0, 1)$ of traders "pay attention" on dates 2 and 3.

Crucial assumption: traders that do not pay attention choose not to trade – the rational choice.
They are "out of the market," holding onto their initial positions.

Rational Benchmark: Solution

Let's solve the model after the disclosure.

In the second trading period, "attentive" investors solve

$$\begin{aligned}
&\arg\max_{D_{A2}} \mathbb{E}\left[-\exp\left(-\rho D_{A1}\left(P_2 - P_1\right) - \rho D_{A2}\left(\tilde{v} - P_2\right)\right) | y\right] \\
&= \arg\max_{D_{A2}} \mathbb{E}\left[-\exp\left(-\rho D_{A2}\left(\tilde{v} - P_2\right)\right) | y\right] \\
&= \arg\max_{D_{A2}} -\exp\left(-\rho D_{A2}\left(\mathbb{E}\left[\tilde{v}|y\right] - P_2\right) + \frac{\rho^2}{2} D_{A2}^2 \mathbb{V}\left[\tilde{v}|y\right]\right).
\end{aligned}$$

We obtain

$$D_{A2} = \frac{\mathbb{E}\left[\tilde{v}|y\right] - P_2}{\gamma \mathbb{V}\left[\tilde{v}|y\right]}.$$

Again, inattentive investors hold onto their initial positions – call them D_{I1}.

Rational Benchmark: Solution

The market clearing condition is

$$(1-\lambda) D_{I1} + \lambda D_{A2} = Q$$

$$\Leftrightarrow P_2 = \underbrace{\mathbb{E}\left[\tilde{v}|y\right]}_{\text{reaction to disclosure}} - \underbrace{\gamma \mathbb{V}\left[\tilde{v}|y\right] \frac{Q-(1-\lambda) D_{I1}}{\lambda}}_{\text{constant risk-premium term}}.$$

- Key result: the response to the disclosure and the ERC

$$\frac{\partial}{\partial y}\mathbb{E}\left[\tilde{v}|y\right] = \frac{\tau_\eta}{\tau_\eta + \tau_v}$$

are not functions of λ.

- The fraction of investors "paying attention" is irrelevant to the pricing of the disclosure.

Rational Benchmark: Drift

Future expected dollar returns conditional on a signal outcome y are:

$$\begin{aligned}\text{Returns after a signal } y &= \mathbb{E}[\tilde{v} - P_2(y)|y] \\ &= \mathbb{E}\left[\tilde{v}|y\right] - \mathbb{E}\left[\tilde{v}|y\right] + \text{risk premium} \\ &= \text{risk premium.}\end{aligned}$$

Since future returns do not depend on the disclosed signal y, there is no drift.

Technical caveat: when looking at *percent* returns, due to the effect in LLV discussed last class, there will be negative drift (“reversal”), but, in a full multi-asset model, this is driven entirely by how news affects future beta. So, it should not show up after controlling for future beta.

Rational Benchmark: Intuition

- Why? Only the beliefs of investors who are actually trading enter the "weighted average belief" in price.
  - Suppose just two investors are in the market and trading.
  - As long as both have processed the disclosure, the price they transact at will reflect its value implications.
- Inattentive investors lose nothing by not paying attention.
  - Prices efficiently reflect the value implications of the disclosure $\Rightarrow$ no gains to trading.
- These results hinge on the assumption that the signal is easily processed by investors who choose to "pay attention" to what is going on in a stock.
- Could perhaps argue this applies to earnings surprises, suggesting that it is challenging to explain PEAD using a "pure inattention" story.

Inattention and Disclosure: Exercise

$$(1-\lambda) D_{I1} + \lambda D_{A2} = Q$$

$$\Leftrightarrow P_2 = \underbrace{\mathbb{E}[\tilde{v}|y]}_{\text{reaction to disclosure}} - \underbrace{\gamma \mathbb{V}[\tilde{v}|y] \frac{Q - (1-\lambda) D_{I1}}{\lambda}}_{\text{constant risk-premium term}}.$$

Would the addition of noise traders affect whether inattention leads to drift?

Inattention: Behavioral Extensions

Behavioral Models, Processing, and Drift

Hirshleifer and Teoh (2003) and DellaVigna and Pollet (2009)

- Highly-cited papers with models where inattentive traders trade in the market even though they either do not process news or misprocess it

- While they are attentive enough to trade, they are not attentive to the signal
  - In recent years, unclear whether a significant amount of capital is bought or sold without knowing the earnings surprise
  - Even for retail, many online brokers show EA coverage when submitting orders

- They fail to rationally account for their ignorance of the signal; justified as follows:

  "we assume that individuals do not fully discount for their imperfect attention informing expectations. Without this assumption, an individual who knew he was inattentive to a relevant information item could, for example, choose not to trade."

  "the same constraints on processing power and memory that make it hard to attend to an aspect of the environment also make it hard to compensate optimally for the failure to attend to an item."

  "He may inattentively fail to reason sufficiently about why the market price differs from his own valuation."

Behavioral Models, Processing, and Drift

Because inattentive traders are now active in the market, prices reflect a weighted average of their beliefs and those of attentive traders

- Recall general formula from last class: when investor i believes that $\tilde{v} \sim N(m_i, \tau_{v,i}^{-1})$,

$$P = \frac{\int \tau_{v,i} m_i di}{\int \tau_{v,i} di} - \gamma Q \left[\int \tau_{v,i} di \right]^{-1}.$$

- Inattentive investors believe $\tilde{v} \sim N(m, \tau_v^{-1})$, attentive investors believe $\tilde{v} \sim N\left(\frac{\tau_v m + \tau_\eta y}{\tau_v + \tau_\eta}, (\tau_v + \tau_\eta)^{-1} \right)$. Substituting, we obtain

$$\frac{\int \tau_{v,i} m_i di}{\int \tau_{v,i} di} = \frac{\lambda (\tau_v + \tau_\eta) \frac{\tau_v m + \tau_\eta y}{\tau_v + \tau_\eta} + (1 - \lambda) \tau_v m}{\lambda (\tau_v + \tau_\eta) + (1 - \lambda) \tau_v} = \frac{\tau_v m + \lambda \tau_\eta y}{\tau_v + \lambda \tau_\eta}$$

and

$$\left[\int \tau_{v,i} di \right]^{-1} = \frac{1}{\lambda (\tau_v + \tau_\eta) + (1 - \lambda) \tau_v}.$$

Behavioral Models, Processing, and Drift

Hence, we have

$$P_2 = \frac{\tau_v m + \lambda \tau_\eta y}{\tau_v + \lambda \tau_\eta} - \gamma Q \frac{1}{\lambda (\tau_v + \tau_\eta) + (1 - \lambda) \tau_v},$$

and thus

$$\frac{\partial P_2}{\partial y} = \frac{\lambda \tau_\eta}{\tau_v + \lambda \tau_\eta} < \frac{\tau_\eta}{\tau_v + \tau_\eta}. \tag{1}$$

Price reaction to earnings is lower than under the rational benchmark.

This naturally leads to PEAD:

$$\begin{aligned} \text{Returns after a signal } y &= \mathbb{E}[\tilde{v} - P_2(y) | y] \\ &= \frac{\tau_v m + \tau_\eta y}{\tau_v + \tau_\eta} - \frac{\tau_v m + \lambda \tau_\eta y}{\tau_v + \lambda \tau_\eta} + \text{risk premium.} \end{aligned}$$

Applying equation (1), this increases in y.

Behavioral Models, Processing, and Drift

Inattentive investors lose money to attentive ones.

- They *buy* after bad news and *sell* after good news, behaving as "contrarians," while attentive investors do the opposite, behaving as "trend chasers."
  - Given that good news is not fully priced, selling after good news is the wrong choice.

- To see this, notice that inattentive investor demand is

$$\frac{\tau_v \times \mathbb{E}[v - P]}{\gamma} \propto \mathbb{E}[v - P] = m - \frac{\tau_v m + \lambda \tau_\eta y}{\tau_v + \lambda \tau_\eta} + \frac{\gamma Q}{\lambda\left(\tau_v + \tau_\eta\right) + (1 - \lambda)\,\tau_v}.$$

  This *decreases* in y.

$\Rightarrow$ Attentive investor demand must *increase* in y for the market to clear.

Other Behavioral Explanations

Alternative behavioral explanations, where PEAD could arise even when most active investors *do* observe earnings surprises:

- Disposition effect: traders sell after good news to lock in gains and do not sell after bad news to avoid realizing losses
- Sticky beliefs: investors' beliefs may systematically underreact to new info
  - This tendency may be amplified when traders are distracted, which could explain the "Friday" effect
  - Coibion and Gorodnichenko, 2015, a highly-cited economics paper, provides broad evidence among forecasters

Rational Learning

- Learning: investors' beliefs may seemingly underreact to new info, but investors had no way to know this at the time
  - Banerjee et al.'s discussion of Blankespoor et al. review calls this "unknowable relevance"
  - Under this theory, the magnitude of PEAD should decline over time as traders learn (consistent with empirical evidence, e.g., Martineau, 2022)
  - Bayesian interpretation: investors start with a prior over the correct response to earnings and gradually adjust as data reveals this prior is too low
  - However, Marrow and Nagel (2024) considers the decision problem of a naive Bayesian investor who learns over time from anomalies; rate of decay and evidence that anomalies decline around publication is seemingly inconsistent with rational, efficient Bayesian learning

Acquisition and Integration Costs

Acquisition and Integration: Overview

Let's next consider disclosures that are expensive to obtain and/or whose value implications require expertise and time.

- The signals investors glean by processing such disclosures are effectively private information; can be analyzed using the Grossman-Stiglitz-Hellwig framework
- Like other costly private signals, hard-to-process disclosures cannot be perfectly priced in equilibrium or there would be no incentive to process them in the first place
  - In equilibrium, either all investors process the disclosure or all investors are indifferent between processing or not processing
  - As long as they account for their info disadvantage when trading by "learning from the price," the investors not processing disclosures in equilibrium are behaving rationally
  - However, as we will see, processing costs still affect overall investor welfare
- To see the impact of greater processing costs on pricing/drift, we will slightly modify Goldstein and Yang's model from last class

Acquisition and Integration: Model

Formally, suppose now that:

1. Following Goldstein-Yang: continuum of price-taking CARA investors, stock payoffs $\tilde{v} \sim N\left(0, \tau_v^{-1}\right)$, per-capita stock supply Q, bond with return normalized to 1 in unlimited supply

2. No public signal; instead, investors can acquire a disclosure at cost c_F.

3. If perfectly "integrated" by an investor, the disclosure would reveal $\tilde{v}$
   common simplifying assumption, not key to results
   can think of $\tilde{v}$ as one component of firm value revealed by the disclosure
   - However, integration requires costly effort τ_ε
   - Given their effort τ_ε, they observe a noisy signal $\tilde{s}_i = \tilde{v} + \tilde{\varepsilon}_i$ where $\tilde{\varepsilon}_i$ has precision τ_ε and is independent across traders
   - The cost they incur is $\frac{k}{2}\tau_\varepsilon^2$, increasing in the precision of the signal they obtain
   - Fits precisely into Goldstein and Yang's setup

4. c_F and k capture acquisition and integration costs, respectively.

Acquisition and Integration: Pricing and Drift

As in slides 1, but now without $\tilde{y}$:

Conjecture an equilibrium in which price is linear in $\tilde{v}, \tilde{x}$:

$$P = p_0 + p_v \tilde{v} + p_x \tilde{x}.$$

Thus, $\tilde{s}_p \equiv \frac{P - p_0}{p_v} = \tilde{v} + \frac{p_x}{p_v} \tilde{x}$.

- $\rho = \frac{p_v}{p_x}$ captures the informativeness of price.
- Then, $\tilde{s}_p$ is a truth-plus-noise signal with precision $\rho^2 \tau_x$.
- So, applying Bayes' rule for normals,

$$E\left[\tilde{v} | \tilde{s}_i, \tilde{P}\right] = \frac{\tau_\varepsilon \tilde{s}_i + \rho^2 \tau_x \tilde{s}_p}{\tau_v + \tau_\varepsilon + \rho^2 \tau_x};$$

$$Var\left[\tilde{v} | \tilde{s}_i, \tilde{P}\right] = \frac{1}{\tau_v + \tau_\varepsilon + \rho^2 \tau_x}.$$

Acquisition and Integration: Pricing and Drift

Prices reflect a weighted average of informed and uninformed beliefs:

$$\frac{\int \tau_{v,i} m_i di}{\int \tau_{v,i} di} = \frac{\mu\left(\tau_v + \tau_\varepsilon + \rho^2\tau_x\right)\frac{\tau_\varepsilon \int s_i di + \rho^2 \tau_x s_p}{\tau_v + \tau_\varepsilon + \rho^2\tau_x} + (1-\mu)\left(\tau_v + \rho^2\tau_x\right)\frac{\rho^2\tau_x s_p}{\tau_v + \rho^2\tau_x}}{\mu\left(\tau_v + \tau_\varepsilon + \rho^2\tau_x\right) + (1-\mu)\left(\tau_v + \rho^2\tau_x\right)}$$

$$= \frac{\rho^2\tau_x s_p + \mu\tau_\varepsilon v}{\tau_v + \rho^2\tau_x + \mu\tau_\varepsilon};$$

$$\left[\int \tau_{v,i} di\right]^{-1} = \frac{1}{\tau_v + \rho^2\tau_x + \mu\tau_\varepsilon}.$$

Hence, given that the supply of the asset after noise traders buy x shares is $Q - x$,

$$P = \frac{\rho^2\tau_x \tilde{s}_p + \mu\tau_\varepsilon \tilde{v}}{\tau_v + \rho^2\tau_x + \mu\tau_\varepsilon} - \gamma(Q - \tilde{x})\frac{1}{\tau_v + \rho^2\tau_x + \mu\tau_\varepsilon}$$

Following the same steps as in last class (“matching coefficients”), we obtain $\rho = \frac{\mu\tau_\varepsilon}{\gamma}$ in equilibrium.

Acquisition and Integration: Pricing and Drift

$$P = \frac{\rho^2 \tau_x \tilde{s}_p + \mu \tau_\varepsilon \tilde{v}}{\tau_v + \rho^2 \tau_x + \mu \tau_\varepsilon} - \gamma(Q - \tilde{x}) \frac{1}{\tau_v + \rho^2 \tau_x + \mu \tau_\varepsilon}$$

Implications:

- Note if the signal were fully processed by all traders, we would just have $P = v$.
- The slope coefficient on v is

$$\frac{\partial P}{\partial v} = \frac{\rho^2 \tau_x + \mu \tau_\varepsilon}{\tau_v + \rho^2 \tau_x + \mu \tau_\varepsilon} < 1,$$

  which implies v is not fully priced
- When investors in equilibrium better process the disclosure ($\mu \uparrow$ or $\tau_\varepsilon \uparrow$), $\frac{\partial P}{\partial v}$ increases through two channels:
  - Direct channel: fixing price informativeness ρ, $\frac{\partial P}{\partial v}$ goes up
  - Indirect channel: when μ or τ_ε increase,
    - price informativeness $\rho = \frac{\mu \tau_\varepsilon}{\gamma} \uparrow$

    $\Rightarrow$ the price weight on s_p, and thus v, $\uparrow$

Acquisition and Integration: Pricing and Drift

$$P = \frac{\rho^2 \tau_x \tilde{s}_p + \mu \tau_\varepsilon \tilde{v}}{\tau_v + \rho^2 \tau_x + \mu \tau_\varepsilon} - \gamma(Q - \tilde{x}) \frac{1}{\tau_v + \rho^2 \tau_x + \mu \tau_\varepsilon}$$

Implications:

- Drift in the direction of the disclosure, if it were perfectly processed, $\tilde{v}$:

$$\begin{aligned} \mathbb{E}[\tilde{v} - P | v] &= v - \mathbb{E}\left[\frac{\rho^2 \tau_x \tilde{s}_p + \mu \tau_\varepsilon \tilde{v}}{\tau_v + \rho^2 \tau_x + \mu \tau_\varepsilon} \bigg| v \right] + \text{risk premium} \\ &= \underbrace{v \left(1 - \frac{\rho^2 \tau_x + \mu \tau_\varepsilon}{\tau_v + \rho^2 \tau_x + \mu \tau_\varepsilon} \right)}_{>0} + \text{ risk premium} \end{aligned}$$

  Similarly, each informed investor expects price to drift in the direction of her private signal s_i

- But, this is only empirically detectable if we can measure v / investors' private information
- Might argue that the constructs a researcher can measure (e.g., line items on the financial statements) are easy to process for large investors in more recent years
  most have access to large datasets, data scientists
- However, complex transformations (e.g., machine learning) could enable measuring private information based on public disclosures, especially when looking historically

Acquisition and Integration: Equilibrium Choices

Next, let's step back and consider the equilibrium when traders decide whether to pay acquisition and integration costs.

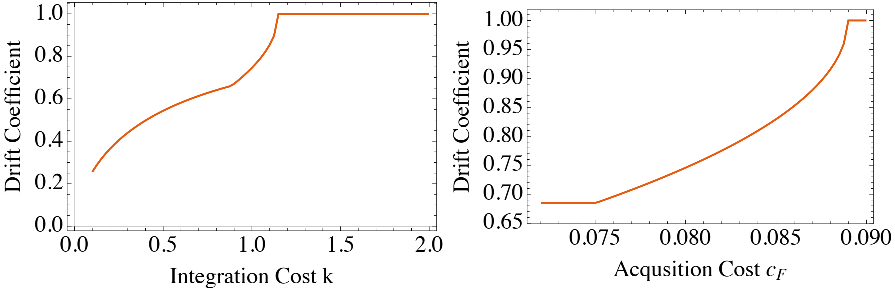
- There is no equilibrium in which disclosure is perfectly processed, i.e., $\mu = 1$ and $\tau_\varepsilon = \infty$
- As in Slides 1, μ and τ_ε satisfy a two-dimensional system of equations in equilibrium:

Condition 1 Traders who acquire the disclosure choose the optimal amount of integration τ_ε^*

Condition 2 Either (i) all traders acquire/integrate the disclosure and the benefits to doing so strictly exceed the costs, or (ii) traders are indifferent between acquiring and integrating the disclosure or not

Formal comparative statics require applying the multivariate implicit function theorem; I will apply numerical analysis

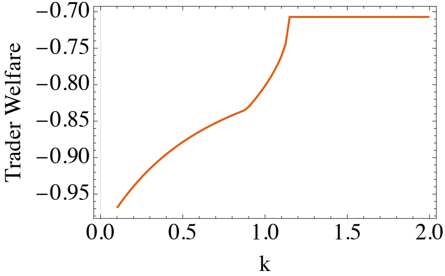
Processing Costs and Drift

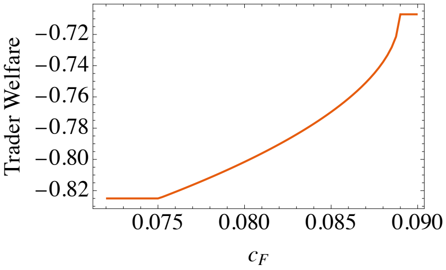


Greater processing costs lead to higher drift (perhaps unsurprisingly).

Relevant to literature on shocks to processing costs (e.g. EDGAR).

Processing Costs and Welfare





Yet they lead to *lower* welfare!

Intuitively, better info raises the price traders must pay for shares and raises liquidity, reducing their ability to exploit noise traders.

Will discuss welfare effects more in future classes.

Rational Inattention Literature

- "Rational inattention" models in economics often take a different approach to modeling information acquisition costs

- They think of attention as a finite resource to be allocated across tasks (e.g., Sims, 2003; Veldkamp, 2011)
  - Typically modelled via entropy, a more general way to capture informativeness than signal precisions

- Leads to a "budget constraint" on attention but no explicit cost
  - The Lagrange multiplier on this constraint serves as a shadow cost, leading to similar results

Summary: Information Processing and Drift

Core message: disclosure can be public yet still only gradually priced if attention or processing is limited, but only under certain conditions.

Three cases:

1. Rational inattention, easily-processed disclosure: inattentive investors optimally choose not to trade, and attentive traders fully price the news, so no drift.
   - Inattention does not harm traders.
2. Behavioral inattention, easily-processed disclosure: when inattentive traders remain active and fail to account for their ignorance, dampened reactions and drift emerge.
   - Inattention harms traders.
3. Costly-to-process disclosure: effectively generates private signals; equilibrium pricing is incomplete, and drift can be rational.
   - Lower processing costs lead to less drift but perhaps surprisingly harms investors (but benefits noise traders).

Outline

Ex Ante Cost of Capital

Recap

- Last class, we showed that in one-period CARA-normal models, prices take the form:

$$P = E[\tilde{v}] - \gamma Var[\tilde{v}]\, Q.$$

or in an N-asset economy:

$$P_i = E[\tilde{v}_i] - \gamma Cov\left[\tilde{v}_i, \sum_{j=1}^{N} \tilde{v}_j\right] Q = E[\tilde{v}_i] - \rho Cov\left[\tilde{v}_i, \tilde{M}\right].$$

- Information can be incorporated as follows:

$$\begin{aligned} P(\tilde{s}) &= E[\tilde{v}|\tilde{s}] - \gamma Var[\tilde{v}|\tilde{s}]\, Q; \\ P_i(\tilde{s}_i) &= E[\tilde{v}_i|\tilde{s}_i] - \gamma Cov\left[\tilde{v}_i, \sum_{j=1}^{N} \tilde{v}_j|\tilde{s}_i\right] Q. \end{aligned}$$

On average, information has no impact on the expected cash flow component of price, but reduces the risk premium component.

Extending the ideas to a dynamic world

Our analysis thus far has focused on “single period” settings and considered how disclosure quality affects either:

Expected price following the disclosure: $E\left[P\left(\tilde{s}\right)\right]$, or

Expected returns following the disclosure: $E\left[\frac{\tilde{v}-P\left(\tilde{s}\right)}{P\left(\tilde{s}\right)}|\tilde{s}\right]$.

- But, of course, investors trade all the time, and there are sporadic/periodic disclosures.
- The insights do not really change as long as one focuses on how *historical* information affects *current* expected returns.
- But, there are two concerns...

Extending the ideas to a dynamic world

- First, many times researchers examine prices following the passage of a regulation but prior to the release of new information

  e.g., following the adoption of IFRS (Daske, Hail, Leuz, and Verdi (2008))

  – How does knowledge that better information will arrive in the future affect prices today? Need a dynamic model!

- Second, disclosure quality is likely *persistent* empirically, so that firms with higher quality disclosures in the past are likely also expected to have higher quality disclosures in the future.

  $\Rightarrow$ To understand empirical associations between past info & the cost of capital, we may need to understand how future info affects the cost of capital.

- We will next allow for trade period prior to the release of the disclosure, and ask how future info affects current prices (sometimes called "ex-ante" cost of capital).

- We will assume the information has a systematic component so that it has a "direct effect."

  In particular, we will look at a single-asset model where the asset is in positive supply.

Ex-ante cost of capital

In a standard setting, even under general distributions and (concave) investor utility functions, the anticipation of future information does not affect current prices/expected returns.

Let's look at this in a simple setting where the firm only pays out a single dividend at some future date T and investors are homogeneous.

- Let $\tilde{W}$ denote the investors' possibly random wealth from sources other than their investment in the firm under consideration.
- Note that market clearing requires the investor hold the per-capita supply in each period; let this be 1.
- For the investor to not wish to deviate from holding 1 in date t, they must not be willing to buy and hold an additional unit of the stock at date t. Thus, the following derivative must be zero:

$$\left\{ \frac{\partial}{\partial D} E_t \left[u \left(\tilde{W} + \tilde{v} + D \left(\tilde{v} - P_t \right) \right) \right] \right\}_{D=0} = 0.$$

Ex-ante cost of capital

$$\left\{ \frac{\partial}{\partial D} E_t \left[u\left(\tilde{W} + \tilde{v} + D\left(\tilde{v} - P_t \right) \right) \right] \right\}_{D=0} = 0.$$

This yields:

$$P_t = E_t \left[\frac{u'\left(\tilde{W} + \tilde{v} \right)}{E_t \left(u'\left(\tilde{W} + \tilde{v} \right) \right)} \tilde{v} \right].$$

- Unless the disclosure affects the distribution of $\tilde{W}$ or $\tilde{v}$ (a real effect), information revealed between date t and T is nowhere to be found.
- The intuition is that information does not affect how the asset is allocated, and thus does not affect the holdings of any investor. Moreover, it does not affect the investors' payoffs. Its only effect is to change the time at which uncertainty is resolved.
- This was originally shown in Ross (1989) and studied in a model with private information in Christensen, de la Rosa, and Feltham (2010).

Overlapping Generations

Dutta and Nezlobin (2017): Overview

- This paper analyzes the ex-ante cost of capital in an *overlapping generations setting.*
- A rather extreme assumption, but aims to capture the notion that certain investors have to liquidate their investments and consume today, or have "short-term investment horizons" for some other reason.
  - Short-term horizon captures institutional frictions such as performance chasing in investment funds / learning about asset-manager skill
- **Main finding**: information can influence the ex-ante cost of capital for firms that are growing or shrinking.
  - Note this paper is subject to the one-asset systematic risk pricing concerns.
  - Somewhat hard to interpret certain empirical predictions in light of this.
  - Results apply most clearly to economy-wide changes in disclosure policy.
- Key intuition: with short-term horizons, investors care about intermediate-horizon prices, which are a function of disclosure.
  - See also Clinch (2013, AJM, "Disclosure quality, diversification and the cost of capital").

Dutta and Nezlobin (2017): Assumptions

- Dynamic setting. Investors at date t sell at date $t+1$ to a new set of investors.
- Each period, the firm pays out a dividend/cash flow of $\tilde{c}_t$.
- The firm has a different scale in each period:

$$\tilde{c}_t = k_t * \tilde{x}_t.$$

  This enables them to consider growing/shrinking firms.

- Changes in scale are costly, and these costs are financed via equity issuances.
  - This is not vital to the results but closes the model; we won't focus on investment costs.

Dutta and Nezlobin (2017): Findings

- Firm releases an information signal at date t about $\tilde{x}_{t+1}$:

$$\tilde{S}_t = k_{t-1}\left(\tilde{x}_{t+1} + \tilde{\eta}_t\right)$$

  This has two distinct effects.
  Notice they are careful to keep the signal-to-noise ratio of the signal independent of firm scale

- **Price risk:** investors sell their asset at a price $\tilde{p}_t$
  - This depends upon what investors learn in the next period, $\tilde{S}_{t+1}$
- **Dividend risk:** investors receive a dividend in the next period $\tilde{c}_t$

- These two forces are ubiquitous in models with short-term trading horizons.

Dutta and Nezlobin (2017): Findings

- They examine the impact of changing disclosure quality in the future on present asset prices.
- **Price risk increases in disclosure quality.** More information is impounded into price in the future.
- **Dividend risk decreases in disclosure quality.** We now know more about the dividend to be paid.
- The net effect depends upon the relative magnitude of the price risk and dividend risk effects.
- This is determined by the prominence of dividends in the short versus long term, which comes down to growth.

Dutta and Nezlobin (2017): Multiple Assets

- **Multi-asset extension:** every firm reveals a disclosure.

  $$\tilde{s}_t^j = \tilde{x}_{t+1}^j + \tilde{e}_t^j.$$

  They add (i) a systematic component into all firms' cash flows and (ii) systematic source of error into all firms' signals.

  The latter ensures that the disclosures do not perfectly reveal systematic uncertainty about future cash flows:

  $$\tilde{e}_t^j = \tilde{\eta}_t + \tilde{\upsilon}_t^j.$$

- When they assess the quality of public information in that setting, they adjust $Var\left(\tilde{\eta}_t\right) = \sigma_\eta^2$.

- This corresponds to simultaneously adjusting the information quality of *all* firms, $Var\left(\tilde{s}_t^j | \tilde{x}_{t+1}^j\right)$.

Outline

Disclosure and Beliefs about Risk

- Most of the literature on disclosure in capital markets focuses on disclosure about expected future cash flows.
  - Consider the set up we've been examining where random variables are independent and normal.
  - Posterior variances/covariances are a known constant.
- But, many disclosures discuss risks directly or indirectly.
  - Extreme earnings may revise beliefs about risk upwards.
  - Economic events: lawsuit initiation (risk $\uparrow$), lawsuit resolution (risk $\downarrow$), new risky investment, balance sheet information (e.g., leverage, liquidity), new contracts, short seller reports
  - Climate risk disclosure (regulatory risk, customer demand risk)
  - Compensation disclosure
  - Anything else?
- Classic disclosure models cannot speak to the impact of such disclosures.
  - Key theme: risk disclosure can have different effects than conventional disclosures

Information About Risk Exposures

Heinle and Smith (2017)

This paper takes the standard pricing framework we've been examining and extends it to information on the variance of cash flows.

- Necessary starting point: relax the assumption of a known variance:

$$\tilde{x} = N\left(\mu, \tilde{V}\right) \textit{ conditional} \text{ on } \tilde{V},$$

  where $\tilde{V}$ itself is a random variable. Unconditional distribution is no longer normal!

- Cash-flow variance: $\tilde{V} \sim \textit{Gamma}\,(\alpha, \beta)$:
  - $\Rightarrow$ $E\left(\tilde{V}\right) = \mu_V$; $\textit{Var}\left(\tilde{V}\right) = \sigma_V^2$;
  - $\Rightarrow$ excess kurtosis: $\gamma = 3\frac{\sigma_V^2}{\mu_V^2}$.
- Investors have $U(x) = -\exp(-\rho x)$.

Heinle and Smith (2017)

Notice that we may write:

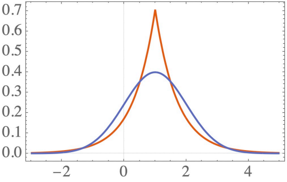
$$
\begin{aligned}
f(x) &= \int f(x,V)dV && \text{defn. of marginal dist.}\\
&= \int f(x|V)f(V)dV && \text{bayes' rule}\\
&= \mathbb{E}\left[f(x|\tilde{V})\right] && \text{defn. of expectation}
\end{aligned}
$$

We can now apply easy-to-use Mathematica functions to obtain and plot the density of $\tilde{x}$:

```
In[385]:= f1[μ_, a_, b_, x_] = Expectation[PDF[NormalDistribution[μ, √V], x], V ≈ GammaDistribution[a, b]];
          f2[μ_, V_, x_] = PDF[NormalDistribution[μ, √V], x];

In[384]:= Plot[{f1[1, 1, 1, xp], f2[1, Mean[GammaDistribution[1, 1]], xp]}, {xp, -3, 5}]
```

Out[384]=



A Note on Mean Uncertainty

- Note mean uncertainty is no different than what we've already analyzed.

- We can reframe an unknown mean as uncertainty about the outcome without deviating from normality:

$$\begin{aligned} \tilde{x} &\sim N\left(\tilde{\mu}, \sigma_x^2\right) \text{ and } \mu \sim N\left(m, \sigma_\mu^2\right) \\ &\quad \text{is equivalent to} \\ \tilde{x} &\sim N\left(m, \sigma^2\right), \text{ where } \sigma^2 = \sigma_x^2 + \sigma_\mu^2. \end{aligned}$$

Heinle and Smith (2017): Price Expression

The firm's equilibrium price satisfies:

$$\begin{aligned} P &= \mu - \rho E\left[Var\left(\tilde{x}|\tilde{V}\right)\right] - VUP_0 \\ &= \mu - \rho Var\left(\tilde{x}\right) - VUP_0, \end{aligned}$$

where line 2 follows by the law of total variance.
Note: the VUP term arises due to kurtosis.

The derivation is simplified by applying the law of iterated expectations (recall it can be used to evaluate expectations over multiple variables one at a time):

$$\begin{aligned} & \frac{\partial}{\partial D} E\left[-\exp\left(-\rho D\left(\tilde{x} - P\right)\right)\right] \\ = \ & \frac{\partial}{\partial D} E\left[-E\left[\exp\left(-\rho D\left(\tilde{x} - P\right)\right)|\tilde{V}\right]\right] \\ = \ & \frac{\partial}{\partial D} E\left[-\exp\left(-\rho D\left(\mu - P\right) + \rho^2 \frac{D^2}{2}\tilde{V}\right)\right]. \end{aligned}$$

Heinle and Smith (2017): Price Expression

$$
\begin{aligned}
& \frac{\partial}{\partial D} E\left[-\exp\left(-\rho D(\mu - P) + \rho \frac{D^2}{2}\tilde{V}\right)\right] \\
= \quad & \frac{\partial}{\partial D} - \exp\left(-\rho D(\mu - P)\right) E\left[\exp\left(\rho \frac{D^2}{2}\tilde{V}\right)\right].
\end{aligned}
$$

Now, $E\left[\exp\left(\rho \frac{D^2}{2}\tilde{V}\right)\right]$ is the MGF of a gamma distribution:

$$
E\left[\exp\left(\rho \frac{D^2}{2}\tilde{V}\right)\right] = \left(1 - \frac{\rho D^2}{2\beta}\right)^{-\alpha}.
$$

So, the first-order condition simplifies to:

$$
\begin{aligned}
0 \quad = \quad & \frac{\partial}{\partial D}\left[-\left(1 - \frac{\rho D^2}{2\beta}\right)^{-\alpha} \exp\left(-\rho D(\mu - P)\right)\right] \\
= \quad & K * \left(2P\beta - 2\beta\mu + 2\alpha D + \mu\rho D^2 - P\rho D^2\right).
\end{aligned}
$$

Heinle and Smith (2017): Assumptions

Introducing disclosure:

- Investors observe $\tilde{S}$, which is the mean of τ draws from a Poisson distribution with parameter equal to the true variance.

- Investors update to a new Gamma distribution; $\tilde{V}|\tilde{S} \sim \mathit{Gamma}$;
  - $E\left(\tilde{V}|\tilde{S}\right) = E\left(\tilde{V}\right) + \frac{Cov\left(\tilde{V},\tilde{S}\right)}{Var\left(\tilde{S}\right)}\left(\tilde{S} - E\left(\tilde{S}\right)\right)$;
  - $Var\left(\tilde{V}|\tilde{S}\right) = Var\left(\tilde{V}\right) - \frac{Cov\left(\tilde{V},\tilde{S}\right)^2}{Var\left(\tilde{S}\right)} + \left(\frac{Cov\left(\tilde{V},\tilde{S}\right)}{Var\left(\tilde{S}\right)}\right)^2 \frac{\tilde{S}-\mu_V}{\tau}$.

- The highlighted term illustrates that the conditional variance is not constant.

Heinle and Smith (2017): CoC and VUP

- Price after disclosure is given by:

$$P\left(\tilde{S}\right) = \mu - RP_0 - \phi\left(\tau\right) VUP_0 - \alpha\left(\tau\right)\left(\tilde{S} - E\left[\tilde{S}\right]\right).$$

- $\phi'\left(\tau\right) < 0$: VUP decreases as signal becomes more precise.

- $\alpha\left(\tau\right)$: “response coefficient” to risk disclosure.

Heinle and Smith (2017): Other Findings

- Disclosure is more informative when it is good news (low risk) and leaves greater uncertainty when it is bad news

- Risk disclosure response coefficient
  - Decreases in expected cash flow variance,
  - Increases in risk aversion and variance uncertainty

- Moreover,
  - *On average* risk disclosure decreases cost of capital, however, disclosure can increase the cost of capital
  - Mean and risk disclosure are substitutes
  - Firms disclose more when their risks are higher than expected
  - In large economies, information about market risk matters

Heinle, Smith, and Verrecchia (2018)

- In this paper, we suppose beta is unknown:

$$\tilde{x} = \mu + \tilde{\beta}\tilde{y},$$

  In this case, $\tilde{\beta}$ can have both a mean and variance effect.

- The mean and variance effects of $\tilde{\beta}$ are correlated, which leads to skewness. Non-normal distribution.

- Investors tend to like skewness, as it puts risk in situations where they are more able to bear it.

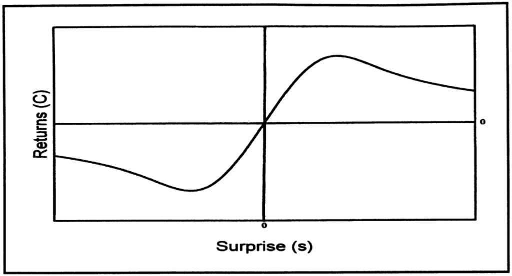
- Disclosure about $\tilde{\beta}$ can reduce skewness and reduce prices on average.

Earnings and Uncertain Risk

Subramanyam (1995)

- This paper analyzed a setting where investors were risk neutral and investors receive a truth-plus-noise signal with unknown precision. Primarily a statistical problem:

FIGURE 1
The Returns-Surprise Relation
The graph depicts the returns function (C) when the precision is distributed in the form of a gamma.



- This type of relationship was empirically documented in Freeman and Tse (1992).

Beyer and Smith (2021)

- Analyzes how investors learn about betas from earnings and the resultant relationships between earnings, prices, and returns.
- Large empirical literature surrounding the relationship between earnings and future returns, e.g., Ball and Brown (1968), Bernard and Thomas (1989).
  - Typically viewed as a delayed reaction, but some question whether risk could play a role.
- Earnings betas work well at estimating future return betas, in fact, better than historical return betas (Nekrasov and Shroff (2009)).
- These differences can partially explain the excess returns to value and size portfolios relative to predictions based upon historical return betas (Baginski and Wahlen (2003), Cohen, Polk, and Vuolteenaho (2009)).
- Empirically: investors appear to be aware of earnings' predictive power for risk; earning announcements are used to learn about betas (Smith and So (2022)).

Beyer and Smith (2021)

- So, what can we get out of modelling this phenomenon?
- Continuum of firms and investors, CARA utility.
- Firm i's terminal cash flows $\tilde{x}_i$ equal:

$$\tilde{x}_i = \tilde{\alpha}_i + \tilde{\beta}_i \tilde{f}_x,$$

  where $\tilde{\alpha}_i \sim N\left(0, \eta_\alpha^{-1}\right)$, $\tilde{\beta}_i \sim N\left(\mu_{\beta,i}, \eta_\beta^{-1}\right)$, and $\tilde{f}_x \sim N\left(0, \eta_x^{-1}\right)$.
  These terms are mutually independent.
- Without loss of generality, we let $\int_0^1 \mu_{\beta,j} dj = 1$.
- Prior to trade, firm i releases an earnings report:

$$\tilde{e}_i = \tilde{\alpha}_i + \tilde{\beta}_i \tilde{f}_e.$$

Beyer and Smith (2021)

- Note, $\tilde{f}_e$ equals "aggregate earnings," $\int_0^1 \tilde{e}_j dj$, and is thus observable.
  - Since firms' earnings are only related through f_e, this means that, when updating on e_i, $\{e_i, f_e\}$ is sufficient for $\{e_j\}_{j\in[0,1]}$.

- We assume aggregate earnings reveal something about aggregate future payoffs: $\tilde{f}_x = \tilde{f}_e + \tilde{f}_p$. Consistent with, e.g., Ball, Sadka, and Sadka (2009).

- Let $\tilde{s}_i = \tilde{e}_i - \mu_{\beta,i}\tilde{f}_e = \tilde{\alpha}_i + (\tilde{\beta}_i - \mu_{\beta,i})\tilde{f}_e$.
  - This looks like an "earnings surprise:" it is actual earnings less what would be expected based on other public information about the macroeconomy.

Beyer and Smith (2021)

- Let's apply the rule that, given X, Y are joint normal,

$$E[X|Y] = E[X] + \frac{Cov[X,Y]}{Var[Y]}(Y - E[Y]).$$

- Setting $X = \tilde{\alpha}_i$ and $Y = \tilde{s}_i$, and treating f_e as known,

$$E\left[\tilde{\alpha}_i | \{\tilde{e}_j\}_{j\in[0,1]}\right] = E\left[\tilde{\alpha}_i | \tilde{s}_i, \tilde{f}_e\right] = \frac{\eta_\beta}{\eta_\beta + \eta_\alpha \tilde{f}_e^2}\tilde{s}_i.$$

  Setting $X = \tilde{\beta}_i$ and $Y = \tilde{s}_i$, and treating f_e as known,

$$E\left[\tilde{\beta}_i | \{\tilde{e}_j\}_{j\in[0,1]}\right] = E\left[\tilde{\beta}_i | \tilde{s}_i, \tilde{f}_e\right] = \mu_{\beta,i} + \frac{\eta_\alpha \tilde{f}_e}{\eta_\beta + \eta_\alpha \tilde{f}_e^2}\tilde{s}_i$$

- Higher earnings lead to higher perceived beta when $\tilde{f}_e > 0$ and lower perceived beta when $\tilde{f}_e < 0$.

- The relative amount of learning about $\tilde{\alpha}_i$ versus $\tilde{\beta}_i$ depends upon $\left|\tilde{f}_e\right|$.

Beyer and Smith (2021)

- With some work, one can show that the pricing equation we found earlier continues to hold: $P_i = E\left[\tilde{x}_i|\tilde{s}_i,\tilde{f}_e\right] - \rho Cov\left[\tilde{x}_i, \int \tilde{x}_j dj|\tilde{s}_i,\tilde{f}_e\right]$.
  Question: why wouldn't this be obvious?

- Note since $\int \tilde{x}_j dj = \tilde{f}_x$, we have:

$$\begin{aligned} P_i &= E\left[\tilde{x}_i|\tilde{s}_i,\tilde{f}_e\right] - \rho Cov\left[\tilde{x}_i,\tilde{f}_x|\tilde{s}_i,\tilde{f}_e\right] \\ &= E\left[\tilde{\alpha}_i|\tilde{s}_i,\tilde{f}_e\right] + E\left[\tilde{\beta}_i|\tilde{s}_i,\tilde{f}_e\right] E\left[\tilde{f}_x|\tilde{f}_e\right] - \rho E\left[\tilde{\beta}_i|\tilde{s}_i,\tilde{f}_e\right] Var\left[\tilde{f}_x|\tilde{f}_e\right], \end{aligned}$$

  where the final step applies the law of total *covariance*, a generalization of the law of total variance.

- Substituting for these variables yields:

$$P_i\left(\mathbf{e}\right) = p_{i0}\left(f_e\right) + p_1\left(f_e\right) \times e_i,$$

  where:

$$p_{i0}\left(\tilde{f}_e\right) = -\frac{\mu_{\beta,i}\eta_\beta}{\eta_\beta + \eta_\alpha f_e^2}\frac{\rho}{\eta_p} \text{ and } p_1\left(f_e\right) = 1 - \frac{f_e\eta_\alpha}{\eta_\beta + \eta_\alpha f_e^2}\frac{\rho}{\eta_p}.$$

Beyer and Smith (2021): ERCs and Macro Performance

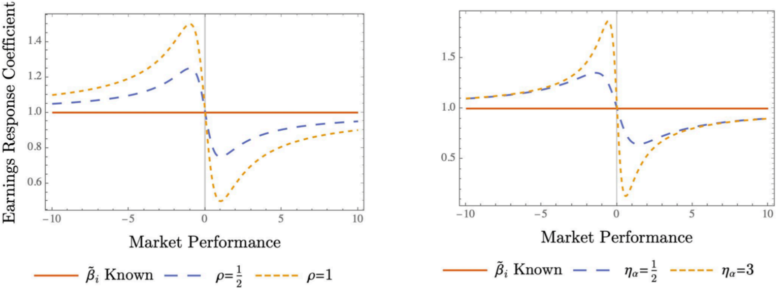


Figure 1: *This figure plots the relationship between the ERC and market performance for various values of investor risk aversion ρ and the precision of firms' idiosyncratic performance η_α. The plots also indicate the benchmark ERC when investors are perfectly informed about a firm's beta (ERC=1).*

Beyer and Smith (2021): Asymmetric Price Response

Proposition. Suppose $\mu_{\beta,i} > 0$. Then, on average, ERCs are higher when earnings are negative than when earnings are positive:

$$\mathbb{E}\left[p_1\left(\tilde{f}_e\right)|e_i < 0\right] > \mathbb{E}\left[p_1\left(\tilde{f}_e\right)|e_i > 0\right].$$

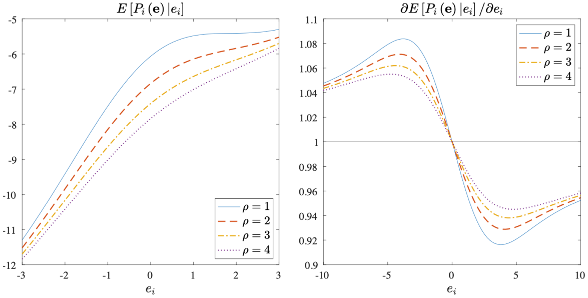


Figure 2: *The figure plots the expected price and the average sensitivity of price to earnings as a function of the firm's earnings for various values of investors' risk-aversion parameter ρ. The graphs illustrates that, on average, prices are more sensitive to changes in earnings when earnings are negative than when they are positive. The parameters held constant in the graph are $\mu_{\beta,i} = 1$, $\eta_\beta = \eta_p = \rho = 2$, and $\eta_\alpha = .5$.*

Beyer and Smith (2021): PEAD and Autocorrelation

Proposition. If current market performance f_e is positive (negative), then:

i. Investors' assessment of firm i's expected exposure to systematic risk increases if and only if $e_i > \mu_{\beta,i} f_e$ ($e_i < \mu_{\beta,i} f_e$);

ii. Firm i's expected returns increase (decrease) in its earnings e_i;

iii. Returns exhibit positive (negative) autocorrelation.

Beyer and Smith (2021): Earnings & Return Volatility

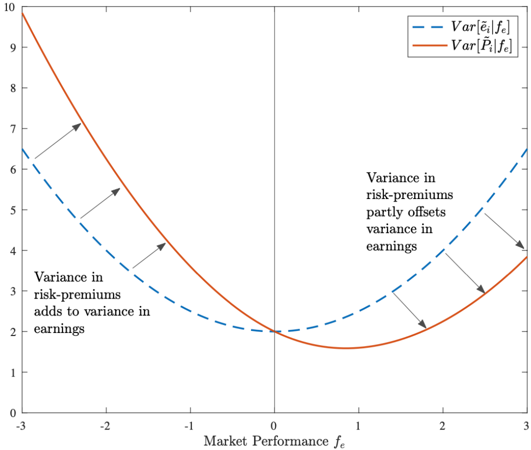


Figure 3: *This figure illustrates the relationship between market performance f_e and both the variances of earnings-induced returns, $Var[\tilde{P}_i|f_e]$, and earnings, $Var[\tilde{e}_i|f_e]$. The parameters held constant in the graph are $\mu_{\beta,i} = 1$, $\eta_\beta = \eta_p = \rho = 2$ and, $\eta_\alpha = .5$.*

Empirical Measurement

Smith and So (2022): A Method for Testing this Theory

- This paper develops a method to assess the risk information in a given information event.
- Many papers use textual analysis to quantify risk information, but it is difficult to decipher text that is news to investors vs. boilerplate text.
- Classic approach to get at new information is to do an event study and look at price reactions.
- But, stock price depend on both mean and risk information. How to disentangle the two?
- We show how this can be done using option prices.
  - Obvious approach: use change in implied volatility surrounding the event.
  - But implied volatility climbs mechanically when information events near.
  - We show this can be addressed by applying an adjustment for "expected announcement volatility."

Background: Black-Scholes Implied Vol

The Black-Scholes formula yields that an option's price O can be expressed as:

$$O = BS\left(P, K, \sigma, t, r\right)$$

where:

- P = current stock price
- K = option strike price
- σ = future volatility of stock price, typically in annualized units (typically between 20% and 100% for individual stocks)
- t = time to maturity
- r = interest rate

Note each of these parameters is directly observable except for σ, so σ can be inverted from the traded price of the option:

$$\sigma_{implied} \quad \text{solves} \quad O_{observed} = BS\left(P, K, \sigma_{implied}, t, r\right).$$

Background: Patell and Wolfson (1979)

Information events temporarily spike volatility. Let $R_{t,\tau}$ denote returns from date t to $t+\tau$. Surrounding an info event, we have:

$$\begin{aligned} var\left[R_{t,\tau}\right] &= \tau \times \text{ “normal daily ret. var”} + \text{excess ret. var created by announcement} \\ &= \tau\sigma^2 + \sigma^2_{ann} \end{aligned}$$

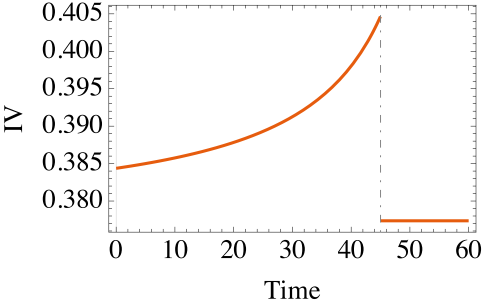
Recall that implied vol is typically presented on an annual basis, so an option as of date t that expires on date $t+\tau$ after the event has:

$$\begin{aligned} \sigma_{implied} &= \sqrt{\frac{365}{\tau} \times var\left[R_{t,\tau}\right]} \\ &= \sqrt{365}\sqrt{\sigma^2 + \frac{1}{\tau}\sigma^2_{ann}}. \end{aligned}$$

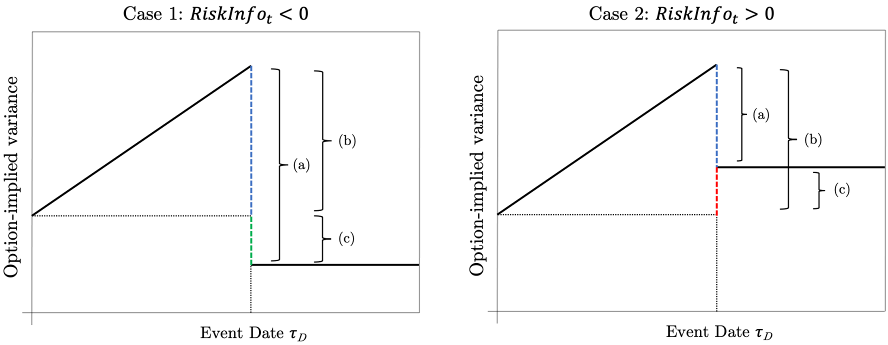
We see that this decreases in τ. Alternatively, fixing the maturity date T, this will increase as time passes (why?).

Background: Patell and Wolfson (1979)

In the following plot, the announcement occurs on day 45:



Smith and So (2022): Intuition



(a) Change in Option-Implied Variance ΔIV_t

(b) Expected Event-Date Variance $\mathbb{E}^{\mathbb{Q}}_{\tau_D}[\tilde{J}^2]$

(c) Risk Information $RiskInfo_t$

Smith and So (2022): Key Findings

- Investors receive information on firms' riskiness equivalent in magnitude to approximately 29% (13%) of the implied variance in firms' stock returns expected over the month (six months) following the announcement.
- Investors can tease apart when the risks discussed in EAs will be resolved and make their way into prices.
- Captures increases in spending on R&D, leverage, and distress risk
- Investors learn about all the Fama-French betas as well as idiosyncratic vol.
- Textual analysis based metrics do a poor job of tracking investor learning about risk; seem to be acknowledging known risks.