Disclosure and Financial Markets Lecture 3: Imperfect Competition

Kevin C. Smith

Stanford University

Agenda

- Kyle (1985)
- Glosten and Milgrom (1985)
- McNichols and Trueman (1994)
- Froot, Scharfstein, and Stein (1992)
- Admati and Pfleiderer (1988)

Review

- Recall that, in the models considered so far:
  - Infinite number of traders $\Rightarrow$ no impact on price.
  - In their optimization problems, price was held fixed.
- For this reason, we used the term “perfect competition.”
- In most markets, investors do have a price impact, especially larger investors. We call models that take this into account models of “imperfect competition.”
- These models allow us to speak about notions of liquidity, bid-ask spreads, price impact, etc.

Kyle (1985): Background

We will focus on the first part of this paper, which is static. The other two parts are dynamic discrete time and continuous time.

- A better reference for the continuous-time model is Back (1991), which is explained in Back's textbook, chapter 24.

Kyle model studies a market with a single informed trader, uninformed traders, and a market maker.[1]

- The value of the asset is $\tilde{x} \sim N\left(\mu, \sigma_x^2\right)$.
- Informed investor knows the realization of the asset's value $\tilde{x}$, and submits a demand D.
- Noise traders submit a random demand $\tilde{z} \sim N\left(0, \sigma_z^2\right)$, and the market maker observes the total order flow $OF \equiv D + \tilde{z}$.

[1] Content for these slides is adapted from Snehal Banerjee and Itay Goldstein's teaching materials.

Kyle (1985): Equilibrium

- The market maker sets the price as the conditional expectation of the asset's value given the order flow $OF = \tilde{D} + \tilde{z}$:

$$P(OF) = E\left(\tilde{x}|OF\right).$$

- Conjecture a price function of the form $P\left(OF\right) = a + \lambda * OF$ and optimal demand of the form $D = d_0 + \beta\tilde{x}$. Note we have:

$$\begin{aligned} P\left(OF\right) = E\left(\tilde{x}|OF\right) &= \mu + \frac{\beta\sigma_x^2}{\beta^2\sigma_x^2 + \sigma_z^2}\left(OF - d_0 - \beta\mu\right) \\ &\equiv \frac{\sigma_z^2}{\beta^2\sigma_x^2 + \sigma_z^2}\mu + \frac{\beta\sigma_x^2}{\beta^2\sigma_x^2 + \sigma_z^2}(OF - d_0). \end{aligned}$$

- The investor solves:

$$\begin{aligned} \overbrace{D\left(\tilde{x}\right)}^{\text{optimal demand given } \tilde{x}} &= \arg\max_D E\left[D * \left(\tilde{x} - a - \lambda * OF\right)|\tilde{x}\right] \\ &= \arg\max_D D * \left(\tilde{x} - a - \lambda * D\right) \\ &= \tfrac{1}{2\lambda}(\tilde{x} - a). \end{aligned}$$

Kyle (1985): Equilibrium

In summary, in equilibrium, we must have that:

$$\begin{aligned} P(OF) &= \frac{\sigma_z^2}{\beta^2\sigma_x^2+\sigma_z^2}\mu + \frac{\beta\sigma_x^2}{\beta^2\sigma_x^2+\sigma_z^2}(OF - d_0); \\ D(x) &= \frac{1}{2\lambda}(x-a). \end{aligned}$$

Thus, to satisfy the equilibrium conjecture, we must have:

$$d_0 = -\frac{1}{2\lambda}a; \quad a = \frac{\sigma_z^2}{\beta^2\sigma_x^2+\sigma_z^2}\mu - d_0; \quad \lambda = \frac{\beta\sigma_x^2}{\beta^2\sigma_x^2+\sigma_z^2}; \quad \beta = \frac{1}{2\lambda}.$$

Solving the system of equations, we arrive at:

$$\begin{aligned} \beta &= \frac{\sigma_z}{\sigma_x} \\ \lambda &= \frac{1}{2\beta} = \frac{\sigma_x}{2\sigma_z} \\ a &= \mu \\ d_0 &= -\frac{\sigma_z}{\sigma_x}\mu. \end{aligned}$$

Kyle (1985): Implications

- "Kyle's λ" is a measure of **market illiquidity** i.e., it is a measure of the **price impact** of a trade.
  - Note, technically, there is no bid-ask spread in this model.
  - Glosten (1991) gives some insight into how to get to a bid-ask spread in this sort of framework via batch orders.
  - Intuition: a marginal order indicates more orders are coming.

- Illiquidity $\lambda = \frac{\sigma_x}{2\sigma_z}$ increases in the amount of uncertainty about the fundamental $\tilde{x}$ and decreases in the amount of noise.

- The informed trader internalizes the effect that she has on the price, and wants to trade less aggressively when λ is large.
  - Hence, we see that β is inversely related to λ.

Kyle (1985): Implications

- The market maker's posterior variance is half the prior variance:

$$Var\left(\tilde{x}|OF\right) = \sigma_x^2 - \frac{\left(\beta\sigma_x^2\right)^2}{\beta^2\sigma_x^2+\sigma_z^2} = \frac{1}{2}\sigma_x^2.$$

  This holds regardless of parameters!

- Intuition: suppose we multiply σ_z by 2.
  - The direct effect is to make order flow a noisier signal of informed demand
  - The indirect effect is to make the informed trader trade more intensively (β increases by a factor of 2)
  - The two effects precisely offset, leaving the informativeness of order flow unchanged

Kyle (1985): Implications

An increase in σ_z (noise) increases profits for the trader:

$$\begin{aligned} E\left[D * (\tilde{x} - P)\right] &= E\left[D\left(\tilde{x} - a - \lambda D\right)\right] \\ &= E\left[\frac{(x-\mu)^2}{4\lambda}\right] \\ &= \sigma_x^2 \times \frac{1}{4\lambda} \\ &= \sigma_x^2 \times \frac{\sigma_z}{2\sigma_x} = \tfrac{1}{2}\sigma_z\sigma_x. \end{aligned}$$

However, it doesn't affect the volatility of the price:

$$\begin{aligned} P &= a + \lambda\left(\beta\tilde{x} + \tilde{z}\right) \\ &= a + \tfrac{1}{2}\tilde{x} + \lambda\tilde{z} \\ \Rightarrow Var\left(P\right) &= \tfrac{1}{4}\sigma_x^2 + \lambda^2\sigma_z^2 = \tfrac{1}{2}\sigma_x^2. \end{aligned}$$

Intuitively, this follows because liquidity increases ($\lambda \downarrow$) and trading aggressiveness increases ($\beta \uparrow$) in σ_z in a manner that precisely cancels.

Kyle (1985): Introducing Public Information

What does the model imply about how public information influences market liquidity, as captured by λ? Insider profits?

Formally, suppose we introduce a signal $\tilde{s} = \tilde{x} + \tilde{\varepsilon}$ prior to trade, where the error satisfies the "usual assumptions:" $\tilde{\varepsilon} \sim N(0, \sigma_\varepsilon^2)$, $\tilde{\varepsilon} \perp \tilde{x}$.

Extensions

Note that there are major simplifications relative to the existing models we've analyzed.

- Only one informed trader.
- Everyone is risk neutral, price is set by market makers whose behavior is not formally modeled.

Many papers consider multi-trader Kyle models.

- Looks a bit like a Cournot model where trader demands are analogous to production; competition among informed traders plays a key role.

Kyle (1989) considers a CARA/normal type model with a finite number of informed investors who consider their price impact.

- This is a richer, but more complicated model of microstructure.
- A few papers in accounting have studied disclosure in this framework, e.g., Lambert, Leuz, and Verrecchia (2012); Caskey, Hughes, and Liu (2015).

Glosten and Milgrom (1985): Background

- Glosten and Milgrom developed an alternative microstructure model that is often used to analyze trading and price formation.
  - Simple, one-period version of this model is often used to model the financial market in more complex settings.
  - The reason is that the model is incredibly tractable, has an intuitive equilibrium, and is easy to build on. But, it is very stylized.

- Consider a risk-neutral market maker who faces informed and uninformed traders sequentially and sets bid and ask prices to break even.

- The value of the asset $\tilde{x} \in \{0, 1\}$ with probabilities $\{1 - \pi, \pi\}$.

- There are two types of risk-neutral agents: informed traders with fraction α, uninformed traders with fraction $1 - \alpha$.

- Traders are restricted to buying / selling one unit and arrive sequentially to the market maker.

Glosten and Milgrom (1985): Assumptions

- The uninformed trader buys with probability ρ and sells with probability $1 - \rho$.

- The informed trader is perfectly informed about the value of the asset.
  - Easily shown she must buy (sell) in any equilibrium when observing $\tilde{x} = 1$ ($\tilde{x} = 0$).
  - This assumption is very easy to generalize to the case in which the informed trader observes an arbitrary signal.

- The market maker is competitive and sets prices to get zero profits in expectation, i.e.,

$$a = E\left[\tilde{x} \middle| \text{buy at } a\right]; \, b = E\left[\tilde{x} \middle| \text{sell at } b\right].$$

Note $a - b$ is the bid-ask spread.

Glosten and Milgrom (1985): Example

- **An example:** Suppose $\tilde{x}$ is zero or one with equal probability ($\pi = \frac{1}{2}$); the uninformed trader is equally likely to sell as to buy ($\rho = \frac{1}{2}$); and the trader is equally likely to be informed as uninformed ($\alpha = \frac{1}{2}$). Two periods.

- **At date 0:** The ask and bid prices can be computed using Bayes' Rule. Let $\tilde{I} = 1$ when the trader is informed and $\tilde{I} = 0$ when they are uninformed.

$$
\begin{aligned}
a &= E\left[\tilde{x} | \text{buy at } a\right] \\
&= \Pr\left(\tilde{x} = 1 | \text{buy at } a\right) \times 1 + \Pr\left(\tilde{x} = 0 | \text{buy at } a\right) \times 0 \\
&= \frac{\Pr\left(x = 1, Buy\right)}{\Pr\left(Buy\right)} \\
&= \frac{\Pr\left(I = 0\right)\Pr\left(x = 1, Buy | I = 0\right) + \Pr\left(I = 1\right)\Pr\left(x = 1, Buy | I = 1\right)}{\Pr\left(I = 0\right)\Pr\left(Buy | I = 0\right) + \Pr\left(I = 1\right)\Pr\left(Buy | I = 1\right)}.
\end{aligned}
$$

Glosten and Milgrom (1985): Example

Let's work on simplifying this further. Since the uninformed trader trades at random, we have that x is independent of her order:

$$\Pr\left(x=1, Buy|I=0\right)=\Pr\left(x=1\right)\Pr\left(Buy|I=0\right)=\pi\times\rho.$$

On the other hand, since the informed trader buys only when $x=1$, we have:

$$\Pr\left(x=1, Buy|I=1\right)=\Pr\left(x=1\right)=\pi.$$

Putting things together, we have:

$$\frac{\Pr\left(I=0\right)\Pr\left(x=1, Buy|I=0\right)+\Pr\left(I=1\right)\Pr\left(x=1, Buy|I=1\right)}{\Pr\left(I=0\right)\Pr\left(Buy|I=0\right)+\Pr\left(I=1\right)\Pr\left(Buy|I=1\right)}$$
$$=\frac{(1-\alpha)\rho\pi+\alpha\pi}{(1-\alpha)\rho+\alpha\pi}=\frac{3}{4}.$$

Similarly, we obtain (verify this):

$$\begin{aligned} b &= E\left[\tilde{x}|\text{sell at } b\right]=\Pr\left[\tilde{x}=1|\text{sell at } b\right] \\ &= \frac{(1-\alpha)(1-\rho)\pi}{(1-\alpha)(1-\rho)+\alpha(1-\pi)}=\frac{1}{4}. \end{aligned}$$

Glosten and Milgrom (1985): Example

At date 1: The bid-ask spread depends on the trade at date 0. If the first trade was a buy, then $\Pr(\tilde{x} = 1|\text{buy}) = \hat{\pi} = \frac{3}{4}$. We can think of this as a new prior, so all we have to do is substitute into the equations we derived earlier!

$$\begin{aligned} a &= E[\tilde{x}|\text{buy at } a] = \Pr[\tilde{x} = 1|\text{buy at } a] \\ &= \frac{(1-\alpha)\rho\hat{\pi} + \alpha\hat{\pi}}{(1-\alpha)\rho + \alpha\hat{\pi}} = \frac{9}{10}. \\ b &= E[\tilde{x}|\text{sell at } b] = \Pr[\tilde{x} = 1|\text{sell at } b] \\ &= \frac{(1-\alpha)(1-\rho)\hat{\pi}}{(1-\alpha)(1-\rho) + \alpha(1-\hat{\pi})} = \frac{1}{2}. \end{aligned}$$

If the first trade was a sell, then $\Pr(\tilde{x} = 1|\text{sell}) = \hat{\pi} = \frac{1}{4}$ and we can again substitute into the previous equations to get $a = \frac{1}{2};\ b = \frac{1}{10}$.

Note that the price is a martingale, i.e., $P_t = E_t[P_{t+1}]$ (how do we know this?).

Note that a sell in one period and a buy in the other leaves beliefs at the prior.

Glosten and Milgrom (1985): Extensions

Other variations of the Glosten and Milgrom model that you may see in the literature include:

1. The noise trader might not trade at all with some probability.
2. The informed trader might stick around, trading again in future periods.
3. There might be many informed traders and noise traders. Continuum of traders makes this tractable (e.g., Dow, Goldstein, and Guembel 2017).
4. Endogenous information acquisition.

Glosten and Milgrom (1985): Disclosure

What is the impact of accounting information on spreads in the Glosten and Milgrom model?

Consider the single-period variant with $\pi = \rho = \frac{1}{2}$. Suppose the firm releases a disclosure signal $\tilde{s} \in \{s_L, s_H\}$ that satisfies:

$$\Pr(\tilde{s} = s_H | \tilde{x} = 1) = \Pr(\tilde{s} = s_L | \tilde{x} = 0) = \tau > 1/2.$$

Note τ captures the disclosure's "precision," and the assumption that $\tau > 1/2$ is without loss of generality (why?). This information structure is common in the literature.

Steps to solving for disclosure's impact on spreads:

1. What are the posterior probabilities that $\tilde{x} = 1$ given $\tilde{s} = s_L$ and $\tilde{s} = s_H$?
2. What are the spreads given these posteriors?
3. How do they compare to the case without a disclosure?

Glosten and Milgrom (1985): Disclosure

1. What are the posterior probabilities that $\tilde{x} = 1$ given $\tilde{s} = s_L$ and $\tilde{s} = s_H$?

Applying Bayes' rule and the law of total probability, we obtain:

$$
\begin{aligned}
&\Pr(\tilde{x} = 1 | \tilde{s} = s_H) \\
&= \frac{\Pr(\tilde{x} = 1, \tilde{s} = s_H)}{\Pr(\tilde{s} = s_H)} \\
&= \frac{\Pr(\tilde{s} = s_H | \tilde{x} = 1) \times \Pr(\tilde{x} = 1)}{\Pr(\tilde{s} = s_H | \tilde{x} = 1) \times \Pr(\tilde{x} = 1) + \Pr(\tilde{s} = s_H | \tilde{x} = 0) \times \Pr(\tilde{x} = 0)} \\
&= \frac{\tau/2}{\tau/2 + (1-\tau)/2} \\
&= \tau.
\end{aligned}
$$

A symmetric argument shows $\Pr(\tilde{x} = 1 | \tilde{s} = s_L) = 1 - \tau$.

Glosten and Milgrom (1985): Disclosure

2. What are the spreads given these posteriors?

Applying Bayes' rule and the law of total probability, we obtain:

$$a|s_H = \Pr(\tilde{x} = 1|\text{buy at a}, \tilde{s} = s_H) = \frac{\tau/2 \times (1-\alpha) + \tau \times \alpha}{1/2 \times (1-\alpha) + \tau \times \alpha} = \frac{(\alpha+1)\tau}{\alpha(2\tau-1)+1}.$$

Similarly,

$$a|s_L = \Pr(\tilde{x} = 1|\text{buy at a}, \tilde{s} = s_L) = \frac{(\alpha+1)(\tau-1)}{\alpha(2\tau-1)-1}$$

$$b|s_H = \Pr(\tilde{x} = 1|\text{sell at b}, \tilde{s} = s_H) = \frac{\tau(1-\alpha)}{\alpha(1-2\tau)+1}$$

$$b|s_L = \Pr(\tilde{x} = 1|\text{sell at b}, \tilde{s} = s_L) = \frac{(\alpha-1)(\tau-1)}{\alpha(2\tau-1)+1}.$$

So, the spreads given either $\tilde{s} = s_L$ or $\tilde{s} = s_H$ are exactly the same:

$$a|s_L - b|s_L = a|s_H - b|s_H = \frac{4\alpha(\tau-1)\tau}{\alpha^2(1-2\tau)^2-1}.$$

Glosten and Milgrom (1985): Disclosure

3. How do these spreads compare to the case without a disclosure?

$$a|s_L - b|s_L = a|s_H - b|s_H = \frac{4\alpha(\tau - 1)\tau}{\alpha^2(1-2\tau)^2 - 1}.$$

- Without a disclosure, the bid and ask are exactly the same, but with $\tau = 1/2$ (why?).
- Therefore, the difference in spreads given disclosure vs. no disclosure is:

$$\frac{4\alpha(\tau - 1)\tau}{\alpha^2(1-2\tau)^2 - 1} - \frac{4\alpha(\frac{1}{2} - 1)\frac{1}{2}}{\alpha^2(1-2*\frac{1}{2})^2 - 1} = \frac{\alpha\left(1-\alpha^2\right)(1-2\tau)^2}{\alpha^2(1-2\tau)^2 - 1}.$$

  The numerator of this final expression is positive while the denominator is negative. Hence, the expression is negative.

- This tells us that disclosure decreases spreads in this model.
- If you want more practice, try re-doing this analysis for an arbitrary prior π. (Note the problem is no longer symmetric in this case, so the spreads won't be the same given a high vs. low signal.)

McNichols and Trueman (1994): Background

- In the standard Kyle (1985) model, public information about $\tilde{x}$ reduces the insider's profits.
  - Recall we can think of this result essentially as a reduction in σ_x^2.
- McNichols and Trueman (1994) show this might not be true when traders have short-term horizons.
- The reason is that information needs to get into price for traders to make profits. Disclosure impounds information into prices more quickly.
- This can, in turn, incentivize traders to acquire more information.

McNichols and Trueman (1994): Assumptions

- Two periods. Firm's payoff is $\tilde{v}$, investor privately observes $\tilde{\theta} = \tilde{v} + \tilde{\varepsilon}_1$.
- Firm value $\tilde{v} \sim N\left(\bar{v}, \sigma^2\right)$; signal error $\tilde{\varepsilon}_1 \sim N\left(0, \sigma_1^2\right)$. These are further assumed to be independent.
- Noise trade in the first period is $\tilde{u} \sim N\left(0, \sigma_u^2\right)$.
- Informed trader chooses demand x and has to reverse this in the second period. For tractability, noise traders also reverse their positions in the second period. Why?
- Public signal $\tilde{z} = \tilde{v} + \tilde{\varepsilon}_2$ is revealed between first and second periods with prob. p.
- Public signal error $\tilde{\varepsilon}_2 \sim N\left(0, \sigma_2^2\right)$ is independent of $\tilde{v}$, but may covary with private signal error $Cov\left(\tilde{\varepsilon}_1, \tilde{\varepsilon}_2\right) = \sigma_{12} \geq 0$.
  - How can we think about this covariance?

McNichols and Trueman (1994): Proposition 1

Proposition

Let y denote the total order flow at time 1.

- *If there is no disclosure, then $P_2 = P_1$.*
- *If there is a disclosure, then:*

$$
\begin{aligned}
x &= sign\left(b_2\right)\sqrt{\frac{\sigma_u^2}{\sigma^2+\sigma_1^2}}\left(\tilde{\theta}-\bar{v}\right). \\
P_1 &= \bar{v}+\frac{sign\left(b_2\right)\sigma^2}{2\sigma_u\sqrt{\sigma^2+\sigma_1^2}}y \\
P_2 &= \bar{v}+\frac{sign\left(b_2\right)\sigma^2\sqrt{\sigma^2+\sigma_1^2}\left(\sigma_2^2-\sigma_{12}\right)}{\sigma_u\left(2\left(\sigma^2+\sigma_1^2\right)\left(\sigma^2+\sigma_2^2\right)-\left(\sigma^2+\sigma_{12}\right)^2\right)}y \\
&\quad+\frac{\sigma^2\left(2\sigma_1^2+\sigma^2-\sigma_{12}\right)}{2\left(\sigma^2+\sigma_1^2\right)\left(\sigma^2+\sigma_2^2\right)-\left(\sigma^2+\sigma_{12}\right)^2}\left(\tilde{z}-\bar{v}\right),
\end{aligned}
$$

where b_2 is the coefficient on $\tilde{z}$ in the date 2 price.

McNichols and Trueman (1994): Proposition 1

- The term $sign\,(b_2)$ arises as, with two correlated signals A, B, a greater value of signal A holding fixed signal B can *lower* conditional expectations.

- Intuitively, when would this be the case? Suppose signal A has a highly volatile error that is highly correlated with the error in signal B. Then,
  - Because A is very noisy, it is a poor signal of the fundamental.
  - However, A is a useful signal about the error in B.
  - Thus, fixing B, a high value of A says little about the fundamental. However, it implies that the noise in B was likely higher, which is negative news.

- They assume parameters are such that $b_2 > 0$ moving forward.

McNichols and Trueman (1994): Proposition 1

Steps for the (algebra-intensive) proof:

1. Conjecture an equilibrium in which price is linear in both periods.
2. Note that, given their short-term horizon, the investor's payoff given demand x is $x\left(P_2 - P_1\right)$. So, we can solve their problem as in a static model replacing the asset's payoff with date 2 price.
3. Solve for period 1 demand as a function of period 2 coefficients.
4. Solve for period 1 price given period 2 coefficients.
5. Solve for period 2 coefficients given what was learned from period 1 order flow.

A problem set question will ask you to solve a simplified version.

McNichols and Trueman (1994): Observations

1. The informed trader's expected profits increase in p and σ_2^{-2}.

2. In a certain case, the informed trader's profits can decrease in σ_1^{-2}!
   - An increase in σ_1^{-2} makes the trader's signal more relevant to learning about the fundamental, as opposed to the error in the public disclosure.
   - But, the error is sometimes a larger driver of short-term price.

3. It is better for the investor to know the public signal than the firm's true value.

McNichols and Trueman (1994): Propositions 2, 3, 4, 5

- M&T next consider two ex-ante decisions: whether to collect private information, and how correlated this private information is with the public signal.
  - Greater probability of information release $p \Rightarrow$ more info acquired.
  - More precise disclosure $\sigma_2^{-2} \Rightarrow$ more info acquired.

- Likewise, p and σ_2^{-2} increase the desire to acquire information correlated with the public release.

- Greater $p \Rightarrow$ larger (smaller) price reaction prior to (following) the disclosure.

Froot, Scharfstein, and Stein (1992): Overview

- Traders might herd on the same information when they have short-term horizons in a Kyle model.

- The reason is that they need to trade on information that will get into price by the time they sell.

- If they acquire something that other traders also seem to focus on, it is more likely that other traders will impound the information into prices in the near term.
  - Rests on a key assumption, which is that each trader perceives that they will, at least with some probability, acquire/trade on this info before other traders do.

- Traders can end up trading on information that is irrelevant to fundamentals!

- Could be a driver of fixation on earnings.

Admati and Pfleiderer (1988): Overview

- This paper is a canonical paper on market microstructure.
- Attempts to answer the question: what causes volume and volatility to be concentrated during specific time windows?
- Theoretically, their innovation is to endogenize the decisions of liquidity traders regarding when to trade.
- Their main finding is that liquidity traders have an incentive to coordinate, creating concentrated windows of trade.
- Moreover, informed traders also choose to trade at the same time as liquidity traders.

Admati and Pfleiderer (1988): Assumptions

- Dynamic model that terminates at time T. At this time, an asset pays off:

$$\tilde{F} = \bar{F} + \sum_{t=1}^{T} \tilde{\delta}_t.$$

- All other variables in the model are normal with mean zero. In period t, $\tilde{\delta}_t$ is made public.

- There are n_t traders with private information in period t. They observe $\tilde{\delta}_{t+1} + \tilde{\varepsilon}_t$, where $Var\left(\tilde{\varepsilon}_t\right) = \phi_t$.
  - Initially, $\{n_t\}_{t \in \{1,\ldots,T\}}$ is exogenous.

- Traders are risk neutral and a market maker absorbs demand; price is set to the expected value of the firm given the total order flow.

Admati and Pfleiderer (1988): Assumptions

- Two types of liquidity traders, discretionary and non-discretionary.
  - Non-discretionary liquidity traders must trade $\tilde{z}_t$ immediately upon receiving a shock.
  - Discretionary liquidity traders can choose to trade any time between T' and T''; refer to their shock in time T' as $\tilde{Y}^j$. They can only trade at a single date in the main model.

- Let $\tilde{\omega}_t$ denote date t order flow; let $\tilde{\Delta}_t = \left\{\tilde{\delta}_\tau\right\}_{\tau \leq t}$ and $\tilde{\Omega}_t = \{\tilde{\omega}_\tau\}_{\tau \leq t}$ denote histories. Then:

$$P_t = E\left[\tilde{F}|\tilde{\Delta}_t, \tilde{\Omega}_t\right].$$

- They focus on linear equilibria as in Kyle (1985). Specifically,

$$P_t = E\left[\tilde{F}|\tilde{\Delta}_t\right] + \lambda_t \tilde{\omega}_t = \bar{F} + \sum_{\tau \leq t} \delta_\tau + \lambda_t \tilde{\omega}_t.$$

  Note that $\{\omega_\tau\}_{\tau<t}$ are no longer relevant since they were signals about all $\{\delta_\tau\}_{\tau \leq t}$, which are public on date t.

- Solving the model requires extending the Kyle analysis to multiple informed traders.

Admati and Pfleiderer (1988): Main Results

- Let $\tilde{y}_t^j = \tilde{Y}^j$ if the j^{th} discretionary liquidity trader trades in period t and 0 otherwise.
  - Hence, the variance of total liquidity trade in period t is $\Psi_t = Var\left(\sum_{j=1}^{m} \tilde{y}_t^j + \tilde{z}_t\right)$.
- Key result:

Lemma 1. *If the market maker follows a linear pricing strategy, then in equilibrium each informed trader i submits at time t a market order of* $\tilde{x}_t^i = \beta_t^i(\tilde{\delta}_{t+1} + \tilde{\epsilon}_t)$, *where*

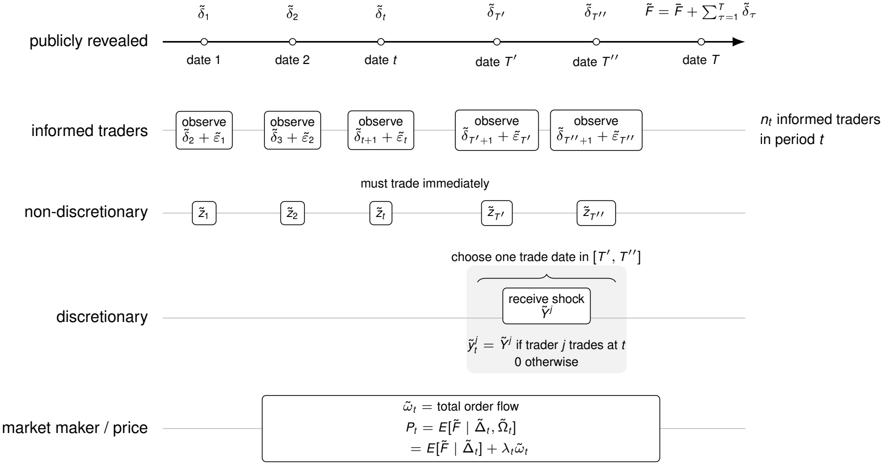
$$\beta_t^i = \sqrt{\frac{\Psi_t}{n_t(\text{var}(\tilde{\delta}_{t+1}) + \phi_t)}} \tag{4}$$

The equilibrium value of λ_t *is given by*

$$\lambda_t = \frac{\text{var}(\tilde{\delta}_{t+1})}{n_t + 1}\sqrt{\frac{n_t}{\Psi_t(\text{var}(\tilde{\delta}_{t+1}) + \phi_t)}} \tag{5}$$

- This result is effectively a generalization of the Kyle analysis to multiple informed traders. It forms the backbone for the remaining analysis.

Admati and Pfleiderer (1988): Model Setup



Admati and Pfleiderer (1988): Main Results

- **Key result of this equilibrium:** price impact on date t, λ_t, decreases in the total variance of noise trade on date t, Ψ_t.

- Moreover, it *decreases* in the number of informed traders n_t on date t as they compete with one another. More recent literature on HFT often analyzes similar comparative statics.

- **Assumption:** liquidity traders take the value of λ_t as a given. This captures the notion that their decisions of when to trade are unobservable.

- Consequently, liquidity traders choose the time of trade t to minimize:

$$E\left[\underbrace{\left(P_t\left(\tilde{\Delta}_t, \tilde{\Omega}_t\right) - \tilde{F}\right)\tilde{Y}^j}_{\text{loss per share} \times \text{shares demanded}} \,\middle|\, \overbrace{\tilde{\Delta}_t, \tilde{\Omega}_{t-1}, \tilde{Y}^j}^{\text{date t information}}\right].$$

Note the trader decides whether to trade before observing date t order flow, which is why this is based on $\tilde{\Omega}_{t-1}$, not $\tilde{\Omega}_t$.

Admati and Pfleiderer (1988): Main Results

Note the demand from traders other than liquidity trader j, $\tilde{\omega}_t - \tilde{Y}^j$, is mean zero and independent of $\tilde{\Delta}_t, \tilde{\Omega}_{t-1}, \tilde{Y}^j$.

- Other liquidity traders' demands are completely random.
- Informed traders' demands depend only upon their private signal in period t, which is independent of past information.

Thus, we can simplify the trader's expected losses as follows:

$$
\begin{aligned}
& E\left[\left(P_t\left(\tilde{\Delta}_t, \tilde{\Omega}_t\right) - \tilde{F}\right)\tilde{Y}^j \mid \tilde{\Delta}_t, \tilde{\Omega}_{t-1}, \tilde{Y}^j\right] \\
=& E\left\{\left[E\left(\tilde{F} \mid \tilde{\Delta}_t\right) + \lambda_t\left(\underbrace{\tilde{\omega}_t - \tilde{Y}^j}_{\text{other traders' order flow}} + \tilde{Y}^j\right) - \tilde{F}\right]\tilde{Y}^j \,\middle|\, \tilde{\Delta}_t, \tilde{\Omega}_{t-1}, \tilde{Y}^j\right\} \\
=& E\left[\left(E\left(\tilde{F} \mid \tilde{\Delta}_t\right) + \lambda_t\tilde{Y}^j - \tilde{F}\right)\tilde{Y}^j \mid \tilde{\Delta}_t, \tilde{\Omega}_{t-1}, \tilde{Y}^j\right]
\end{aligned}
$$

$$\text{since } E\left[\tilde{\omega}_t - \tilde{Y}^j \mid \tilde{\Delta}_t, \tilde{\Omega}_{t-1}, \tilde{Y}^j\right] = 0.$$

Admati and Pfleiderer (1988): Main Results

Continuing,

$$
\begin{aligned}
& E\left[\left(E\left(\tilde{F} \mid \tilde{\Delta}_t\right) + \lambda_t \tilde{Y}^j - \tilde{F}\right) \tilde{Y}^j \mid \tilde{\Delta}_t, \tilde{\Omega}_{t-1}, \tilde{Y}^j\right] \\
&= E\left[\left(E\left(\tilde{F} \mid \tilde{\Delta}_t\right) - \tilde{F}\right) \tilde{Y}^j \mid \tilde{\Delta}_t, \tilde{\Omega}_{t-1}, \tilde{Y}^j\right] + \lambda_t E\left[\left(\tilde{Y}^j\right)^2 \mid \tilde{\Delta}_t, \tilde{\Omega}_{t-1}, \tilde{Y}^j\right] \\
&= \underbrace{E\left[E\left(\tilde{F} \mid \tilde{\Delta}_t\right) - \tilde{F} \mid \tilde{\Delta}_t, \tilde{\Omega}_{t-1}, \tilde{Y}^j\right]}_{\substack{=0 \\ \text{because } \tilde{\Delta}_t \text{ is sufficient} \\ \text{for updating on } \tilde{F}}} \tilde{Y}^j + \lambda_t \underbrace{E\left[\left(\tilde{Y}^j\right)^2 \mid \tilde{\Delta}_t, \tilde{\Omega}_{t-1}, \tilde{Y}^j\right]}_{\substack{=(\tilde{Y}^j)^2 \\ \text{because } \tilde{Y}^j \text{ is known}}} \\
&= \lambda_t \left(\tilde{Y}^j\right)^2.
\end{aligned}
$$

Hence, the liquidity traders simply want to trade when price impact λ_t is the lowest.

Admati and Pfleiderer (1988): Main Results

Now, λ_t is the lowest when the variance of liquidity trade is the highest. So, liquidity traders stick together.

- They choose to trade at the time when there is lowest expected price impact.
- However, analogous to Kyle, price volatility and the information content of price do not change in these periods.

Admati and Pfleiderer (1988): Information Acquisition

- Cost c to learn $\tilde{\delta}_t$ in a particular period.
- Assumption: trader cannot make their presence known.
  - Why does this matter?
- Model has multiple equilibria; they assume maximum info acquisition.
- Informed investors want to learn and trade when there is more liquidity trade.
  - Might this break the result that liquidity traders concentrate?
  - Turns out no: more informed traders in a period actually make liquidity traders better off. (Though best situation is no informed trade at all.)
  - Again, results from competition.
  - In sum, liquidity traders' ability to strategically coordinate on specific times to trade benefits them at the expense of informed traders.

Admati and Pfleiderer (1988): Implications for Trade around Earnings

- What might this model have to say about volume, volatility, and informed trade around earnings?

- Krinsky and Lee (1996): spreads go up around earnings releases.
  - Likely the precisions of informed traders' signals rise around earnings.
  - Thus, A&P framework suggests that these are periods *discretionary* liquidity traders would avoid (somewhat outside our discussion to this point).
  - Yet, volume spikes around these announcements.
    - If there are just *more* informed traders around earnings, this could explain it.
    - Disagreement is another explanation

- Another explanation is that there are more *non-discretionary* noise traders around earnings.
  - Interpreting non-discretionary noise traders as unsophisticated traders, earnings might capture their attention.