Disclosure and Financial Markets Lecture 4: Disclosure, Welfare, and Trade Part 1

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Agenda

1. **Disclosure and welfare in “pure exchange”**
   - Summary/review of early work on risk sharing in financial markets
   - Hirshleifer (1971)
   - Hakansson, Kunkel, Ohlson (1981); Milgrom and Stokey (1982)
   - Diamond (1985)

2. **Why regulate disclosure?**
   - Fishman and Hagerty (1989)
   - Admati and Pfleiderer (2000)

Disclosure and Welfare: Introduction

- Disclosure causes the cost of capital & price vol. to fall on average, liquidity & price efficiency to rise. But are investors **better off** with more disclosure?

- This is a question of welfare.
  - Cannot be fully addressed in the models discussed so far: we haven't modeled noise traders' utility functions!
  - Also important to note that we are ignoring disclosure's role in affecting investment (more on this later).

- To deal with this: prior work instead assumes noise comes from endowments Diamond (1985), Wang (1994), Ganguli and Yang (2009)
  - Implies that investors must trade to arrive at an "efficient allocation" of the stock.
  - Renders the model more difficult to work with; we won't go through the actual algebra.

Efficient Risk Sharing

- **Early work in economics:** optimal allocation of a risky asset among investors involves each investor holding a stake in the asset.
  - First welfare theorem: investors can arrive at such an allocation via trade in a competitive market.
- The stake they hold depends upon their risk preferences, outside exposures, and subjective beliefs.
  - The optimal holding ensures that investors' marginal utilities in each possible state of the world are proportional.
  - The coefficient of proportionality depends upon investors' initial wealths.
- This is covered in Back Chapter 4.
- The arguments for efficient risk sharing apply in the models we've been looking at. But, the first welfare theorem need not, given the presence of heterogeneous information and the potential for delay in trade...

The “Hirshleifer” Effect

- **Hirshleifer (1971):** when investors want to share risk, **“pre-trade”** information has a *negative* impact on welfare.
  - **Intuition:** resolves risk prior to the ability of traders to share this risk.

- **Classic example:** public disclosure of who possesses a disease prior to the writing of an insurance contract.
  - Once it is known who has a disease, it is too late to insure. Ex ante (before knowing if you are sick), everyone is worse off.

- **In financial markets:** the Hirshleifer effect is partially muted...

Hakansson et al. (1981); Ohlson (1984)

- Suppose (i) markets are complete and (ii) agents can trade in a stock with value $\tilde{x}$ **prior to** an information release $\tilde{s}$.
  - Allows them to arrive at an initial Pareto-efficient allocation of the stock.
  - May apply to financial markets where we have frequent trading, and so investors can share risk prior to the release of information.

- **"No trade theorem."** Suppose $f_i\left(\tilde{s}|\tilde{x}\right) = f_j\left(\tilde{s}|\tilde{x}\right)$ and $f_i(\tilde{s}) = f_j(\tilde{s})$. The information release does not lead to trade and does not affect welfare.
  - No welfare decline because investors have already traded to the optimum and agree on how to interpret the disclosure.
  - By revealed preference, no trade $\Rightarrow$ no welfare improvement (in pure exchange models) either.
  - This only mitigates the potential downsides of information in financial markets.
  - Milgrom and Stokey (1984) extend this to the observation of private information signals absent noise trade.

Recall: complete markets means (in static models) that, for any state of the world, there's a linear combination of the traded securities that pays off only in that state. What really matters is that investors can find securities that let them trade on any risk exposure they might want to.

Hakansson et al. (1981); Ohlson (1984)

This does not mean the Hirshleifer effect is unimportant in financial markets.

- Investors may not be able to trade to an efficient allocation prior to information release.
  - Traders may enter the market sporadically (e.g., after getting a job).
  - Example: suppose accounting information perfectly revealed all future cash flows. No stock market by the time you get your academic pay check.

- **More broadly:** The Hirshleifer effect generally does arise when there is trade after disclosure, as it reduces the welfare gains available from such trade.
  - In practice, investors do trade frequently, including after disclosures.
    There are many drivers of this (beyond just traders sporadically entering the market).
  - Trade in secondary markets is essentially always pre-empted by financial disclosures.

- Markets may be incomplete for various reasons. Investors may not be able to fully hedge/trade on certain exposures in stocks.

Disclosure and Welfare: Other Effects in Pure Exchange

Even remaining in the pure exchange framework, disclosure may have other effects on welfare.

1. A simple force: information can enhance **subjective welfare** if traders **disagree** about how to interpret it.
   - Technically, they must have different views on $f(\tilde{s}|\tilde{x})$ or $f(\tilde{s})$; subjective welfare refers to welfare as measured based on individuals' beliefs.
   - Whether we should care about this is a philosophical question: if a regulator ignores this channel, they are engaging in **"paternalism"**
2. Improved risk sharing via **enhanced market completeness** in dynamic settings (Christensen and Qin (2013))
3. A "taste" for information. For example, **non-EU preferences** (Epstein-Zin, ambiguity aversion)
4. Two additional effects in Diamond (1985)...

Diamond (1985): Overview

Diamond (1985) shows, in a Grossman and Stiglitz framework, that the effect of disclosure is more subtle when one takes into account opportunities to acquire private information.

Two additional effects

1. *Disclosure reduces information acquisition costs.*
   - Investors **replicate** each other's costly efforts to get the same **news**.
   - By discouraging information acquisition, disclosure eliminates this inefficient replication.
   - Amplified by the fact that it might be cheaper for the firm than for investors to produce information.

Diamond (1985): Overview

2. *Disclosure can enhance efficient risk sharing by reducing incentives to trade on private information.*

Basic idea: The Hirshleifer effect also applies to privately-held information.

- Private information concentrates the stock in the hands of investors who have favorable signals.
- This means that investors, as a whole, are suboptimally diversified, which reduces welfare.
- While private info plausibly also improves informed investors' welfare by garnering them trading profits, this effect is "zero sum" across investors.

By discouraging information acquisition, disclosure attenuates this negative effect, i.e., causes the average investor to be better diversified.

Diamond (1985): Assumptions

- Starts with a Grossman-Stiglitz (1980) style information-acquisition equilibrium, albeit, in a Hellwig (1980) type model (independent errors in investors' private signals).
- Standard CARA assumptions (risk tolerance $r \Leftrightarrow$ risk aversion $\frac{1}{r}$), bond in unlimited supply with return normalized to 1, independence of random variables.
- The stock pays off $\tilde{u} \sim N\left(Y_0, h_0^{-1}\right)$.
- Traders can acquire a signal at a cost c: $\tilde{y}_t = \tilde{u} + \tilde{\varepsilon}_t$, $\tilde{\varepsilon}_t \sim N\left(0, s^{-1}\right)$.
- T traders with independent endowments $\tilde{x}_t \sim N(0, T * V)$; he then lets T grow large.
  - A common alternative to noise trade; the effect is equivalent here.
  - Letting the variance grow with T is a weird "trick." Ensures that the per-capita supply $\frac{1}{T} \lim_{T \to \infty} \sum_{t=1}^{T} \tilde{x}_t \neq 0$.
  - More common approach is to allow $\tilde{x}_t$ to have a systematic component (Wang (1994), Schneider (2008), Smith (2019)). Becomes much more burdensome.

Diamond (1985): Equilibrium Derivation

- Let λ denote the fraction of informed traders. They first analyze the market *given* a level of λ.
  - Follows just as in our study of Hellwig (1980) in Slides 1.
  - Investors sell their endowments in the market.
  - Only difference is that the total supply now equals the supply that comes from traders' endowments, rather than noise trade. But, this doesn't change anything.

- The next step is to determine the equilibrium level of λ. This requires equalizing informed and uninformed traders' utilities:

  $$E\left[-\exp\left(-r^{-1}D^{I}\left(\tilde{y}_t, P\right)\left(\tilde{u}-P\right)-c\right)\right]=E\left[-\exp\left(-r^{-1}D^{U}\left(P\right)\left(\tilde{u}-P\right)\right)\right]$$

  Just as in Grossman and Stiglitz, we have a unique equilibrium $\lambda^{*}\in[0,1]$.
  question: why is $\lambda=1$ *possible here?*

- See the appendix for more details on how to solve for investors' expected utilities.

Diamond (1985): Equilibrium Derivation

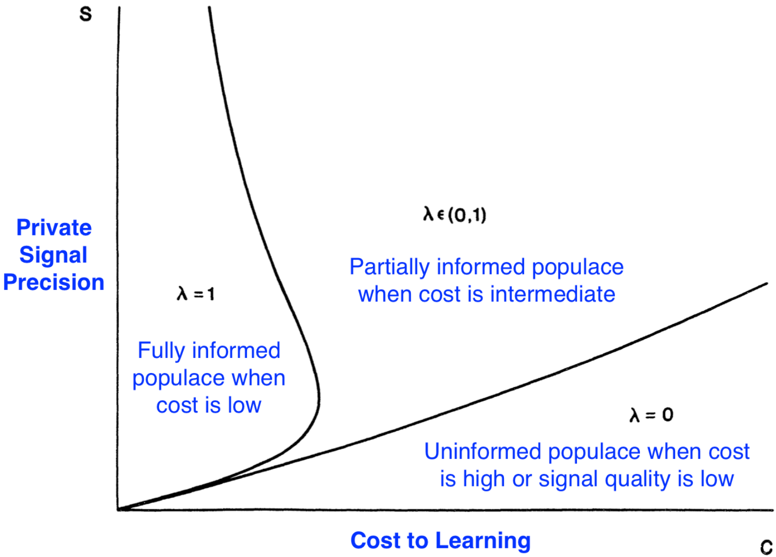


Figure 1. Determinants of λ; The Fraction of Informed Traders

Diamond (1985): Disclosure and Info Acq. Costs

- Diamond shows that introducing a disclosure $\tilde{Y} = \tilde{u} + \tilde{\xi}$ where $Var^{-1}\left(\tilde{\xi}\right) = \Delta$ has the same effect on traders' utilities as increasing the prior precision by Δ.

- He then shows that $\Delta = \frac{s}{\exp\left(\frac{2c}{r} - 1\right)}$ is sufficient to ensure no trader acquires information.

- The firm can produce this information at a negligible cost relative to the market, as they only have to produce it once. So, traders don't have to incur these costs.

- As a result, when disclosure is cheap for the firm, it is generally optimal to choose $\Delta = \frac{s}{\exp\left(\frac{2c}{r} - 1\right)}$.

Diamond (1985): Disclosure and Risk Sharing

- Diamond next partials out the effect of disclosure on information acquisition costs to show that disclosure has an additional effect beyond reducing costs.
  - This additional effect can be positive or negative.

- There is a trade-off between two effects. First, it causes risk to be borne prior to investors being able to trade to arrive at a Pareto optimum.
  - Recall: investors begin with (inefficient) heterogeneous endowments. Hence, disclosure creates a standard Hirshleifer-type effect.

- At the same time, it prevents traders from acquiring private info and holding speculative positions that are suboptimal from an ex-ante risk-sharing perspective.
  - Ex ante: ability to acquire private information is socially detrimental to all traders in pure exchange.
  - But, from an individual trader's perspective, it is optimal to acquire information. Can think of this as a prisoner's dilemma.

- Which effect dominates is a nuanced function of the underlying parameters.
  - "First best" outcome: some binding mechanism to prevent private information acquisition without having to disclose.

Disclosure, Investment, and Welfare

- Disclosure can also affect welfare by **influencing investment decisions.**

- For example: a reduction in the cost of capital can **stimulate productivity/economic growth** as managers become more willing to take on positive expected value projects.
  - Endogenizing investment can add nuance to the relationship between disclosure and the cost of capital (Gao (2010), Cheynel (2013)).
  - In some cases: disclosure can *improve welfare*, and at the same time *raise the cost of capital* by raising risky investment.
  - This effect operates through the risk premium, and so it applies to economy-wide disclosure quality.
  - Beyond investors: risky investment may benefit consumers (i.e., consumer surplus). Existing models tend to ignore this.

- This line of reasoning rests on investors observing the nature of firms' investments.
  - In some cases, the characteristics of firms' investment may be unobservable to the public.
  - Direct disclosure of these characteristics could create proprietary costs.

Disclosure, Investment, and Welfare

- If investments are not observable, managers could act myopically, neglecting value-enhancing investments if they detract from short-term performance indicators. Another body of work abstracts from risk effects to focus on this.

- Alternatively, higher-quality disclosure may expedite the rate at which these indicators reflect the value-impact of unobservable investment.
  - Could stimulate long-term investment.

- In a few classes, we will examine work by Stein (1989), which embeds both of these features.

Disclosure Regulation

- Clearly, documenting welfare benefits to better public info in these models is not sufficient to warrant regulation.

- Analyzing disclosure regulation requires understanding what firms do in an unregulated environment – which we will need models of discretionary disclosure to address (as in Verrecchia (1983) and Dye (1985)).
  - Mandatory disclosure could simply cause firms to reduce their voluntary disclosure, and thus have no, or even a negative impact on overall information quality ("crowding out").

- Formal analysis of voluntary disclosure in models of trade is hard because it leads to "truncated/non-normal" distributions
  - We address this in Banerjee, Marinovic, and Smith (2022) by using recent techniques for solving models in this case.
  - But, we can't address welfare due to tractability issues.

Disclosure Regulation

Arguments for disclosure regulation often fall into one of two categories:

1. Regulation may enable firms to commit to *(truthfully)* disclosing information independent of whether it is positive or negative news. This may enable them to enhance the quality of information they provide.
2. Disclosure may create some form of externality. We will look at two examples today.

Can Diamond's cost argument be thought of as an externality / justify regulation?
Unclear from existing work! Some of my own speculation:

- If investors started out as fully diversified, they would vote for disclosure to avoid wasteful info acquisition in the future.
- But, reality is more complex. Investors who have no control may engage in information acquisition. Controlling interests may not internalize their welfare.

Standardization likely also has benefits: there is value to a common language, which individual firms may not internalize (Hu and Wu (2025)).

Fishman and Hagerty (1989): Overview

- Fishman and Hagerty (1989) start with the assumption that disclosure complements private information acquisition.
  - This opposes the perspective taken by Diamond.
  - Disclosure of this sort would *increase* aggregate info. acq. costs, which is outside their model.

- Complementary disclosure raises a firm's price efficiency at the cost of other firms' price efficiency (negative externality).
  - Investors have limited attention/information processing constraints.
  - Providing better information draws investors' attention away from other firms. This, in turn, raises the efficiency of prices.

- Firm benefits via more efficient stock prices through an investment channel.
  - This specific channel is not crucial to the story; really, there just needs to be some reason the firm cares about its price efficiency.

Fishman and Hagerty (1989): Model

- Economy of I firms.
- Manager of firm I chooses investment level $\bar{\theta}_i$ at convex cost $C(\cdot)$. Given this investment, firm's cash flows $\tilde{\theta}_i \sim N\left(\bar{\theta}_i, h_\theta^{-1}\right)$.
  - Importantly, this investment is not observable.
  - Investment has no impact on variance (strong assumption here).
- Firm releases a disclosure $\tilde{s}_i = \tilde{\theta}_i + \tilde{\varepsilon}_i$.
  - By default, $Var^{-1}(\tilde{\varepsilon}_i) = \tau_L$.
  - The manager can incur a cost of $D(\tau_H) - D(\tau_L)$ to increase precision from τ_L to τ_H. Observable to all.
- Investment and disclosure decisions are made simultaneously.
- Investors can learn about exactly one firm's disclosure.

Fishman and Hagerty (1989): Model

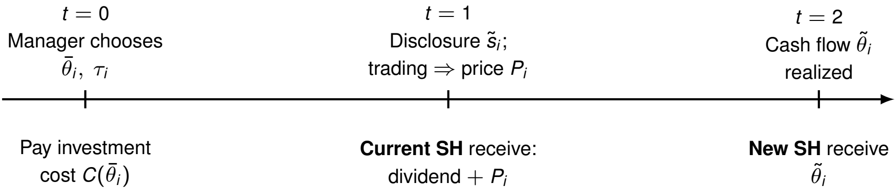
- Investors trade alongside a noise trader in each firm's stock against a competitive market maker as in Kyle (1985).

- Noise traders submit demands $\tilde{z}_i \sim N\left(0, h_z^{-1}\right)$ in firm i that are independent of other variables in the model.

- There is a surplus of cash in the firm paid out as a dividend.
  - Note the dividend is directly reduced by the investment cost.
  - Paper assumes away learning from the dividend by assuming initial cash level unknown (unusual and informal assumption).

Fishman and Hagerty (1989): Manager's Objective

Current shareholders sell the firm at $t = 1$; manager maximizes their total payoff $=$ dividend $+$ price received.



Manager's problem:

$$\max_{\bar{\theta}_i} \quad \underbrace{E[P_i(\tilde{s}_i)]}_{\substack{\text{sale price received} \\ \text{by current SH at } t=1}} \quad - \quad \underbrace{C(\bar{\theta}_i)}_{\substack{\text{reduces dividend} \\ \text{one-for-one}}}$$

- Cash already in the firm is a constant, so it drops out of the argmax.
- Equity-based compensation is one natural way to generate this objective.

Fishman and Hagerty (1989): Intuition for Underinvestment

- The manager's cost and benefit of $\bar{\theta}_i$ are **asymmetric**:
  - **Cost:** $C(\bar{\theta}_i)$ comes straight out of the dividend; manager internalizes full marginal cost.
  - **Benefit:** enters only via $E[P_i]$, which reflects $\bar{\theta}_i$ only *partially*.

- Why is the price response partial?
  - Disclosure $\tilde{s}_i = \bar{\theta}_i + \tilde{\varepsilon}_i$ is noisy.
  - Noise traders $\tilde{z}_i$ further garble order flow.
  - Market maker therefore puts weight $\lambda_{1i} n_i \beta_{1i} < 1$ on each unit of $\bar{\theta}_i$ (shown in the next few slides).

$\Rightarrow$ manager **underinvests** relative to the first-best $\bar{\theta}^*$ (where $C'(\bar{\theta}^*) = 1$).

- Current shareholders *want* an efficient $\bar{\theta}_i$ so new SH pay more, but $\bar{\theta}_i$ is unobservable, so the manager cannot **commit** to $\bar{\theta}^*$.
  - Greater price efficiency $\Rightarrow$ tighter link between $\bar{\theta}_i$ and $P_i \Rightarrow$ less underinvestment: this is the channel through which disclosure creates value.

Fishman and Hagerty (1989): Trading Equilibrium

LEMMA 1: *There is a unique symmetric, linear equilibrium with*

$$X_i(s_i) = \beta_{0i} + \beta_{1i} s_i, \tag{1}$$

$$P_i(n_i x_i + z_i) = \lambda_{0i} + \lambda_{1i}(n_i x_i + z_i), \tag{2}$$

where

$$\beta_{1i} = \left\{\frac{\tau_i h_\theta}{n_i h_z(\tau_i + h_\theta)}\right\}^{1/2},$$

$$\beta_{0i} = -\beta_{1i}\bar{\theta}_i^c,$$

$$\lambda_{0i} = \bar{\theta}_i^c,$$

$$\lambda_{1i} = \frac{1}{n_i + 1}\left\{\frac{\tau_i n_i h_z}{h_\theta(\tau_i + h_\theta)}\right\}^{1/2}.$$

Proof: See Appendix.

n_i is the number of traders who choose to learn about the firm's disclosure.

Fishman and Hagerty (1989): Investment

- Optimal investment choice solves:

$$
\begin{aligned}
& \arg\max_{\bar{\theta}_i} E\left[P_i\left(n_i\left(X_i\left(\tilde{s}_i\right)+\tilde{z}_i\right)\right)\right]-C\left(\bar{\theta}_i\right) \\
= \quad & \arg\max_{\bar{\theta}_i} E\left[\lambda_{0i}+\lambda_{1i}\left(n_i\left(\beta_{0i}+\beta_{1i}\tilde{s}_i\right)+\tilde{z}_i\right)\right]-C\left(\bar{\theta}_i\right) \\
= \quad & \arg\max_{\bar{\theta}_i} \lambda_{1i} n_i \beta_{1i} \bar{\theta}_i - C\left(\bar{\theta}_i\right) \\
\Longrightarrow \quad & C'\left(\bar{\theta}_i\right)=\lambda_{1i} n_i \beta_{1i}.
\end{aligned}
$$

- Now, applying Lemma 1, we can show:

$$
\lambda_{1i} n_i \beta_{1i} = \frac{\tau_i n_i h_\theta}{\tau_i + h_\theta + h_\theta n_i} < 1.
$$

- Note there is a distortion from the “efficient” investment level, $C'\left(\bar{\theta}_i\right)=1$.
- Magnitude of the distortion is $C'^{-1}(1) - C'^{-1}\left(\frac{\tau_i n_i h_\theta}{\tau_i + h_\theta + h_\theta n_i}\right)$, which decreases in n_i and h_θ.

Fishman and Hagerty (1989): Disclosure

- The "socially" optimal level of disclosure trades off disclosure costs against the benefits of price efficiency.

- Starting from this optimal level, an individual firm has an incentive to increase their disclosure quality.
  - They do not internalize the cost of attracting attention away from another firm. This is an externality.

- So, a regulation constraining the ability of firms to convey complementary information may improve welfare.
  - But, it is not clear that many disclosure regulations would have this effect.

Fishman and Hagerty (1989): Discussion

- As previously discussed, disclosure may impact investment through a number of other channels.
  - Managers may possess private information on the quality of their investments.
  - Investment and disclosure might then signal the manager's private information.

- Disclosure costs are highly reduced form. The costs themselves may be related to product-market interactions, which also influence investment.

- Their analysis speaks only to complementary disclosure.
  - What about disclosures that are easy to process, which could push sophisticated investors' attention away? (This is often regulators' viewpoint.)

Admati and Pfleiderer (2000): Background

- A&P show disclosure regulation can be warranted given the following core assumptions:

  (i) firms' values are correlated and the disclosures made by one firm are used by investors in valuing other firms;
  (ii) disclosure of information is costly, and this cost increases in the precision of the disclosed information;
  (iii) information asymmetries between firms and investors reduce firm value.

- This argument holds generally; however, they present a model about a discrete gain to be made from a transaction.
  - Could be thought of as an IPO or large capital infusion.
  - These transactions might be a negative signal and thus inefficiently avoided (lemons problem).
- Dye (1990) presents a similar model in a secondary market setting.

Admati and Pfleiderer (2000): Model

- Two firms with values to their owners of $\tilde{v}_1, \tilde{v}_2$, which are normally distributed. Owners can sell to buyers who value them at $\tilde{v}_i + \delta_i$ where $\delta_i > 0$.
  - Importantly, $Cov\left(\tilde{v}_1, \tilde{v}_2\right) = \rho$.
  - The owners observe but cannot disclose $\tilde{v}_i$.

- Firms release public signals $\tilde{s}_i = \tilde{v}_i + \tilde{\eta}_i$. The noise terms are independent.
  - $Var^{-1}\left(\tilde{\eta}_i\right) = h_i$ is a choice variable; linear cost to the sellers of $\gamma_i h_i$.
  - Signal precision is observed *prior* to the observation of v_i. Why is this important?
  - The signals are released simultaneously.

- After the signals are released, the owner sells to the buyer if and only if the price offered exceeds $\tilde{v}_i$.

Admati and Pfleiderer (2000): Model

- A&P assume that the seller extracts all gains from trade (in a bargaining process, think of this as a take it or leave it offer).

- Thus, if a sale is approved, we have that firm i's price p_i must solve:

$$p_i = E\left[\tilde{v}_i | \tilde{v}_i < p_i, \tilde{s}_1, \tilde{s}_2\right] + \delta_i.$$

  Fairly standard "lemons" problem.

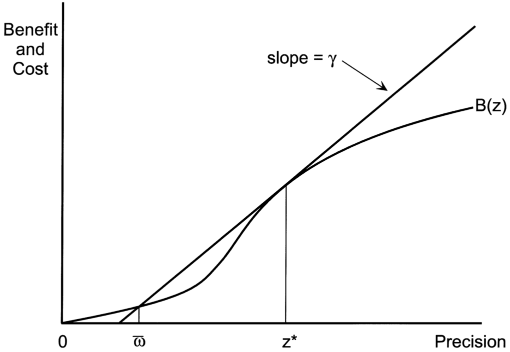
- Let $z = Var^{-1}\left(\tilde{v}_i | \tilde{s}_1, \tilde{s}_2\right)$. The likelihood of a sale can be reduced to $\Phi\left(x\sqrt{z}\right)$, where x is the solution to:

$$x = \delta - \frac{\phi\left(x\sqrt{z}\right)}{\Phi\left(x\sqrt{z}\right)\sqrt{z}}.$$

- Greater precision $\Rightarrow$ greater likelihood of a sale.

Admati and Pfleiderer (2000): Model

- Benefit to a precision choice z,[1] $B(z)$, is equal to δ times the likelihood of a sale.
- It is convex at zero, which leads to a complex expression for the equilibrium choice of z.



[1] Note they work in terms of sum of prior + signal precision, so the minimum choice of z is the prior precision.

Admati and Pfleiderer (2000): Model

- Convexity in the benefit function can lead to multiple equilibria.
  - It can create a strategic complementarity in investment in precision.
  - Regulation requiring a minimum threshold of precision can ensure that the high precision equilibrium is the *only* equilibrium.

- The overall analysis of optimal regulation is complex and parameter sensitive.

Appendix:

Mathematical details on information acquisition in CARA/normal Models

Information Acquisition Equilibrium

Recall that the information acquisition equilibrium solves:

$$E\left[-\exp\left(-r^{-1}D^{I}\left(\tilde{y}_t, P\right)\left(\tilde{u}-P\right)-r^{-1}c\right)\right]=E\left[-\exp\left(-r^{-1}D^{U}\left(P\right)\left(\tilde{u}-P\right)\right)\right]$$

- To derive the information acquisition equilibrium, we need to solve for traders' expected utilities.
- This appendix details how to actually solve for these expected utilities and the resulting equilibrium condition on λ.
- I will do this in the standard Grossman and Stiglitz model, as it is more complex in the uncertain endowments setting. Set endowments to zero; assume any noise comes from noise trade.
  Investor's endowment is correlated with their beliefs in that setting, which leads to an algebraic mess.

Expected Utility Calculations

Let's take a look at calculating the informed trader's EU.

Recall $D^I(\tilde{y}_t, P) = r\frac{E(\tilde{u}|\tilde{y}_t, P) - P}{Var(\tilde{u}|\tilde{y}_t, P)}$. Now condition the LHS on $\tilde{y}_t$ and P to get:

$$E\left\{E\left[-\exp\left(-r^{-1}\frac{E(\tilde{u}|\tilde{y}_t, P) - P}{Var(\tilde{u}|\tilde{y}_t, P)}(\tilde{u} - P) + r^{-1}c\right)|\tilde{y}_t, P\right]\right\}$$

$$= E\left\{-\exp\left[-r^{-1}\frac{E(\tilde{u}|\tilde{y}_t, P) - P}{rVar(\tilde{u}|\tilde{y}_t, P)}(E(\tilde{u}|\tilde{y}_t, P) - P)\right.\right.$$

$$\left.\left.+\frac{1}{2r^2}\left(r\frac{E(\tilde{u}|\tilde{y}_t, P) - P}{Var(\tilde{u}|\tilde{y}_t, P)}\right)^2 Var(\tilde{u}|\tilde{y}_t, P) + r^{-1}c\right]\right\}$$

Expected Utility Calculations

Simplifying, this equals:

$$E\left[-\exp\left(-\frac{1}{2}\underbrace{\left(\frac{E\left(\tilde{u}|\tilde{y}_t,P\right)-P}{\sqrt{Var\left(\tilde{u}|\tilde{y}_t,P\right)}}\right)^2}_{\text{squared normal r.v.}}+r^{-1}c\right)\right]\equiv EU_I.$$

Simplifying this further requires calculating the MGF of a squared normal random variable.

Following the same steps, we obtain that uninformed investors' expected utility:

$$E\left[-\exp\left(-\frac{1}{2}\left(\frac{E\left(\tilde{u}|P\right)-P}{\sqrt{Var\left(\tilde{u}|P\right)}}\right)^2\right)\right]\equiv EU_U.$$

That is, we have the same expression but we no longer condition on $\tilde{y}_t$.

I will now show that EU_I/EU_U has a very elegant form.

Expected Utility Calculations

Step 1: iterated expectations.

$$E\left[-\exp\left(-\frac{1}{2}\left(\frac{E\left(\tilde{u}|\tilde{y}_t,P\right)-P}{\sqrt{Var\left(\tilde{u}|\tilde{y}_t,P\right)}}\right)^2+r^{-1}c\right)\right]=E\left[E\left[-\exp\left(-\frac{1}{2}\left(\frac{E\left(\tilde{u}|\tilde{y}_t,P\right)-P}{\sqrt{Var\left(\tilde{u}|\tilde{y}_t,P\right)}}\right)^2+r^{-1}c\right)|P\right]\right].$$

Step 2: notice that, conditioning on P, and again applying iterated expectations,

$$\frac{E\left(\tilde{u}|\tilde{y}_t,P\right)-P}{\sqrt{Var\left(\tilde{u}|\tilde{y}_t,P\right)}}\sim N\left(\frac{E\left(\tilde{u}|P\right)-P}{\sqrt{Var\left(\tilde{u}|\tilde{y}_t,P\right)}},\frac{Var\left(E\left(\tilde{u}|\tilde{y}_t,P\right)|P\right)}{Var\left(\tilde{u}|\tilde{y}_t,P\right)}\right)$$

Step 3: apply the formula:

$$x\sim N(\mu,\sigma^2)\Rightarrow E\left[\exp\left(a*x^2\right)\right]=\left(1-2a\sigma^2\right)^{-1/2}\exp\left\{\frac{a\mu^2}{1-2a\sigma^2}\right\}$$

to arrive at:

$$EU_I=E\left[-\left(1+\frac{Var\left(E\left(\tilde{u}|\tilde{y}_t,P\right)|P\right)}{Var\left(\tilde{u}|\tilde{y}_t,P\right)}\right)^{-\frac{1}{2}}\exp\left\{-\frac{1}{2}\left(1+\frac{Var\left(E\left(\tilde{u}|\tilde{y}_t,P\right)|P\right)}{Var\left(\tilde{u}|\tilde{y}_t,P\right)}\right)^{-1}\left(\frac{E\left(\tilde{u}|P\right)-P}{\sqrt{Var\left(\tilde{u}|\tilde{y}_t,P\right)}}\right)^2\right\}\right]$$

Expected Utility Calculations

Step 4: Apply the law of total variance to arrive at:

$$1 + \frac{Var\left(E\left(\tilde{u}|\tilde{y}_t, P\right)|P\right)}{Var\left(\tilde{u}|\tilde{y}_t, P\right)} = \frac{Var\left(\tilde{u}|\tilde{y}_t, P\right) + Var\left(\tilde{u}|P\right) - Var\left(\tilde{u}|\tilde{y}_t, P\right)}{Var\left(\tilde{u}|\tilde{y}_t, P\right)} = \frac{Var\left(\tilde{u}|P\right)}{Var\left(\tilde{u}|\tilde{y}_t, P\right)}.$$

Substituting this into the previous equation, we get:

$$EU_I = -\sqrt{\frac{Var\left(\tilde{u}|\tilde{y}_t, P\right)}{Var\left(\tilde{u}|P\right)}} \exp\left(r^{-1}c\right) E\left[\exp\left\{-\frac{1}{2}\left(\frac{E\left(\tilde{u}|P\right) - P}{\sqrt{Var\left(\tilde{u}|P\right)}}\right)^2\right\}\right]$$

Step 5: Combine this with

$$EU_U = E\left[-\exp\left\{-\frac{1}{2}\left(\frac{E\left(\tilde{u}|P\right) - P}{\sqrt{Var\left(\tilde{u}|P\right)}}\right)^2\right\}\right].$$

to obtain:

$$\frac{EU_I}{EU_U} = \sqrt{\frac{Var\left(\tilde{u}|\tilde{y}_t, P\right)}{Var\left(\tilde{u}|P\right)}} \exp\left(r^{-1}c\right).$$

Information Acquisition Equilibrium

The equilibrium is determined by:

$$
\begin{aligned}
1 = \frac{EU_I}{EU_U} &= \sqrt{\frac{Var\left(\tilde{u}|\tilde{y}_t, P\right)}{Var\left(\tilde{u}|P\right)}} \exp\left(r^{-1}c\right). \\
\Leftrightarrow r^{-1}c &= \frac{1}{2} \underbrace{\ln\left(\frac{Var\left(\tilde{u}|P\right)}{Var\left(\tilde{u}|\tilde{y}_t, P\right)}\right)}_{\text{"value of information"}}.
\end{aligned}
$$

We see that the value of learning is determined by the relative variance of cash flows perceived by an informed and uninformed trader.

We can often solve for the equilibrium λ in closed form based on this equation.