

Disclosure and Financial Markets Lecture 6: Factor Models; Intro to Earnings Manipulation

Kevin C. Smith

Stanford University

Factor models: Overview

- Ross's arbitrage pricing theory starts with a *statistical model* of returns:

$$\tilde{R}_i = \alpha_i + \sum \beta_i \tilde{f}_i + \tilde{\epsilon}_i$$

where the $\tilde{\epsilon}_i$'s are independent. It then shows that, if there are many securities, no arbitrage alone yields (see Back Section 6.4-6.5):

$$E[\tilde{R}_i] - r_f = \sum \beta_i \underbrace{\lambda_i}_{\text{factor premia}} .$$

- Intuitively, if this didn't hold, there would be a long-short portfolio with zero risk that earns positive expected value.
 - Short stocks with $E[\tilde{R}_i] - r_f \leq \sum \beta_i \lambda_i$
 - Long those with $E[\tilde{R}_i] - r_f \geq \sum \beta_i \lambda_i$
- Selecting the weights appropriately yields approximately no risk, if there are enough stocks to long and short. $\tilde{\epsilon}_i$ risks "diversify away" and beta risks on the long and short side cancel out.

Factor models: Overview

- Thus, a factor structure in *realized* returns \implies a factor structure in *expected* returns.
 - This holds even if market prices are driven by sentiment/behavioral factors, as long as there is no arbitrage.
 - Thus, sentiment can only influence prices if it has a systematic component.
- APT rests upon the factor structure being common knowledge, perfect competition, there being many assets, etc.
 - Nevertheless, it is a useful benchmark.
 - It tells us that systematic sources of variation in returns are more likely to affect prices because they are more difficult to arbitrage.
- Note the factors in APT can be chosen without loss of generality to be portfolios of stocks, i.e., one need not even consider other variables such as consumption, inflation rates, etc (see Back pg. 139-141).
 - Intuitively, the true underlying factors can be projected onto the space of stock returns. The residual from this projection is not relevant for pricing equities.

Factor models: Overview

- An important question is then: what factors drive covariation in returns? APT tells us the same factors that drive covariation may drive expected returns.
 - From a practical pricing perspective, this is purely a statistical question.
 - *Note:* APT tells us nothing about the factor risk premia. But, we can estimate these too from the data (i.e., Fama Macbeth procedure).
- Recall well-documented factors: beta, size, BTM, momentum, more recently, investment, profitability, and quality.
- What economic rationale might justify these specific factors having non-zero factor premia?

Factor models: Overview

- Size and/or BTM are natural candidates for priced risk factors, whether the market is efficient or not (Berk (1995)).
 - They proxy for the “true” underlying factors.
 - Note this requires failure of the CAPM (otherwise there would be just one factor, the market return)
- This can be seen by examining price within the APT framework. Suppose $\{\tilde{f}_i\}$ are the underlying orthogonalized factors. Then,

$$\begin{aligned} E[\tilde{R}_i] - r_f &= \sum_i \frac{\text{Cov}(\tilde{f}_i, \tilde{R}_i)}{\text{Var}(\tilde{f}_i)} \lambda_i \\ &\equiv \sum_i \beta_i^* \lambda_i. \end{aligned}$$

Therefore, we have:

$$\text{MktCap}_i = \sum \frac{E[\text{div}_t]}{(r_f + \sum_i \beta_i^* \lambda_i)^t}$$

Factor models: Overview

$$MktCap_i = \sum \frac{E[\text{div}_t]}{(r_f + \sum_i \beta_i^* \lambda_i)^t}$$

- Consider factor j . Firms with large values of β_j^*
 1. Have higher discount rates and thus **lower market caps**;
 2. Have returns that covary through \tilde{f}_j .
- Thus, fixing expected cash flows, firms with larger market caps have smaller values of risk-factor loadings and lower future returns.
- Market cap has a huge amount of noise as it is also driven by scale (which influences the numerator, $E[\text{div}_t]$). BTM normalizes for some of this.
- Stated differently, violation of the CAPM assumptions + the forces of arbitrage are sufficient to create the Fama-French 3-factor model.
- What about profitability/quality?
 - Controlling for MktCap, a firm with higher future dividends *must* have higher returns.

Factor models: Overview

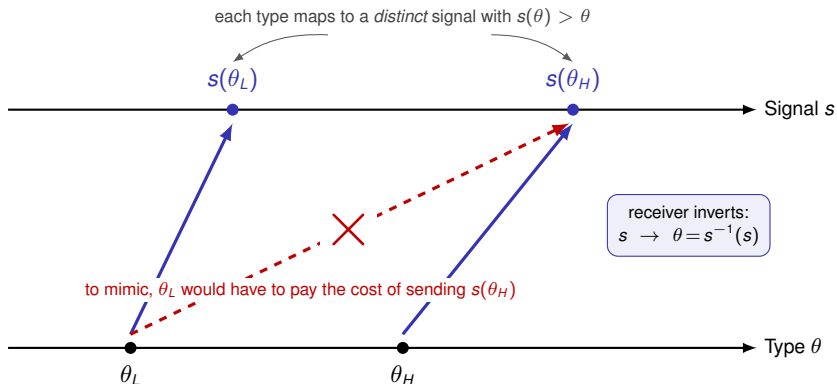
- What are the relevant violations of the CAPM that lead to additional priced factors? Is there an equilibrium theory? Many reasons (though no clear consensus).
 - ICAPM (see Back, Section 10.1, Campbell et al. (2018) and references therein).
 - Small/value stocks may have more negatively skewed or kurtotic systematic components of their payoffs (Harvey and Siddique (2000)).
 - Small/value stocks may have systematic liquidity shocks (Acharya and Pedersen (2005)).
 - Small/value stocks may be subject to systematic negative sentiment (Lee, Shleifer, and Thaler (1991)).
- Remember “small/value” is just a proxy for some unobserved underlying economic risks/mispricing.
 - The causal link is that the underlying risk/mispricing both leads to higher future returns and leads a firm to be categorized as small/value.

Overview: Earnings Manipulation

- In the models we have considered to this point, the signal revealed by the firm is typically out of the firm's control.
 - While papers such as Diamond (1985), Fishman and Hagerty (1989), and Gao and Liang (2013) analyzed the optimal choice of disclosure quality, after choosing this quality, the firm had no ability to impact the actual outcome of the disclosure.
- Thus, the models we have studied do not speak to the discretion that often goes into accounting statements.
- In the next two lectures, we will look at “costly signalling” models in which a firm's manager can influence the outcome of the disclosure directly.
 - These models are signalling models (in the game-theoretic sense), even though they are not always explicitly referred to as such.
 - Some types cannot mimic others because of differential costs to choosing different signals (“single-crossing” condition is often satisfied).

Separating equilibrium: the basic idea

- In a *separating* equilibrium, each type θ sends a *distinct* signal $s(\theta)$, so the receiver can perfectly *invert* the type from the signal.



- Invertibility:** the mapping $\theta \mapsto s(\theta)$ is one-to-one, so observing s pins down θ .
- No mimicking (incentive compatibility):** the cost to θ_L of producing $s(\theta_H)$ exceeds the benefit of being mistaken for the high type.
 - “Single-crossing”: marginal cost of signaling falls in θ , so low types do not want to mimic.

Overview: Earnings Manipulation

Main questions:

- How does manipulation influence stock prices, managers' utility, and uncertainty?
- What drives the amount of manipulation?

Key assumptions:

1. Manipulation activity is not observable.
 - Otherwise, there would be no reason to engage in it.
2. External agents can predict and correct for incentives to manipulate.
 - This does not stop the manager from manipulating.
 - If they were to refrain from manipulating, the market would continue to believe that they were manipulating and adjust the signal as if they were.

Agenda: Earnings Manipulation

1. Static “deterministic” model of manipulation
2. Static “uncertain” model of manipulation
Fischer and Verrecchia (2000)
3. Dynamic deterministic model of manipulation
Stein (1989)
4. Next class: dynamic uncertain model of manipulation
Beyer, Guttman, and Marinovic (2018)

A simple static model

- Consider a manager who runs a firm with value $\tilde{v} \sim N(0, \sigma_v^2)$ and wants to maximize its price.
- The market sets the firm's price to its expected value given publicly available information. "risk-neutral pricing" – a CARA trading model would deliver similar results.
- The manager observes earnings $\tilde{e} = \tilde{v} + \tilde{n}$, where $\tilde{n} \sim N(0, \sigma_n^2)$. They can bias these earnings at a cost $\frac{1}{2}cb^2$, such that the market instead observes:

$$\underbrace{\tilde{r}}_{\text{"reported earnings"}} = \tilde{e} + b.$$

Bias costs could capture reputational or legal costs to getting caught, or even ethics.

- We will examine Bayesian Equilibria (BE): requires that the market **correctly conjectures** the manager's action (the bias) in equilibrium.
 - Could think of this as arising from repeated experience in interpreting firms' earnings.
 - The idea that there is no uncertainty about bias is not realistic. But, it is reasonable to think that seasoned investors know about and adjust for some degree of earnings management.

The Cost to Firms of Cooking the Books

Jonathan M. Karpoff, D. Scott Lee, and Gerald S. Martin*

Abstract

We examine the penalties imposed on the 585 firms targeted by SEC enforcement actions for financial misrepresentation from 1978–2002, which we track through November 15, 2005. The penalties imposed on firms through the legal system average only \$23.5 million per firm. The penalties imposed by the market, in contrast, are huge. Our point estimate of the reputational penalty—which we define as the expected loss in the present value of future cash flows due to lower sales and higher contracting and financing costs—is over 7.5 times the sum of all penalties imposed through the legal and regulatory system. For each dollar that a firm misleadingly inflates its market value, on average, it loses this dollar when its misconduct is revealed, plus an additional \$3.08. Of this additional loss, \$0.36 is due to expected legal penalties and \$2.71 is due to lost reputation. In firms that survive the enforcement process, lost reputation is even greater at \$3.83. In the cross section, the reputation loss is positively related to measures of the firm's reliance on implicit contracts. This evidence belies a widespread belief that financial misrepresentation is disciplined lightly. To the contrary, reputation losses impose substantial penalties for cooking the books.

A simple static model

- Suppose the market conjectures a constant bias of b^* .
(\Rightarrow a separating equilibrium)

$$P = E[\tilde{v}|\tilde{r}] = \frac{\overbrace{\sigma_v^2}^{\text{"ERC"}}}{\sigma_v^2 + \sigma_n^2} \underbrace{(\tilde{r} - b^*)}_{\text{signal is de-biased}}.$$

This follows because, under the conjecture, $E[\tilde{r}] = b^*$.

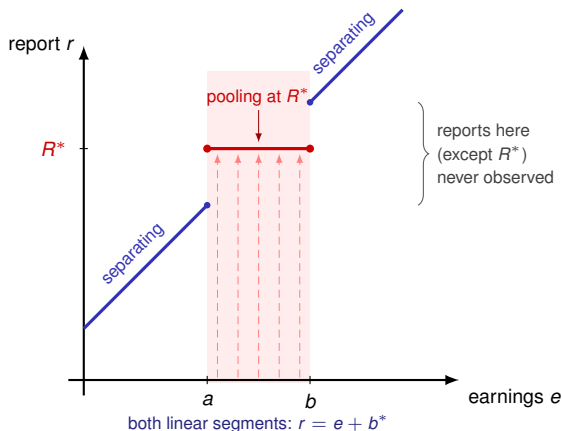
- So, the manager solves:

$$b^{Eq} = \arg \max_b \frac{\sigma_v^2 (\tilde{e} + b - b^*)}{\sigma_v^2 + \sigma_n^2} - \frac{1}{2} cb^2$$
$$\Leftrightarrow b^{Eq} = c^{-1} \frac{\sigma_v^2}{\sigma_v^2 + \sigma_n^2}.$$

- That is, bias is proportional to the marginal weight placed on earnings. It increases in σ_v^2 and decreases in σ_n^2 .
- B.E. requires: $b^{Eq} = b^*$. Bias costs are a deadweight loss, but it is unclear what they capture in the model.

A simple static model

- Guttman, Kadan, and Kandel (2006, TAR) show there are also pooling equilibria.



- These equilibria have certain levels of earnings that are never realized, and are associated with pessimistic off-equilibrium-path beliefs.
 - As such, these equilibria are not robust to signalling refinements like D1.
- There are also non-linear separating equilibria.

Fischer and Verrecchia (2000)

- Fischer and Verrecchia develop a model in which bias is uncertain, adding realism to the effect of manipulation on beliefs.
 - As opposed to backing out true bias, the market backs out expected bias. They get it right on average, but not all the time.
- Generally speaking, the market might not fully grasp the manager's ethics, their incentives, or their perception of the penalties to manipulation.
- Specific implementation: uncertain incentives. The manager maximizes:

$$\tilde{x}P - \frac{1}{2}cb^2,$$

where $\tilde{x} \sim N(\mu_x, \sigma_x^2)$.

- This means the manager sometimes might want to decrease price (normality is necessary for tractability).

Fischer and Verrecchia (2000)

- They examine linear PBE. Start with the market's conjecture of bias for any given realization of $\{\tilde{e}, \tilde{x}\}$, $b(e, x)$. They focus on:

$$\begin{aligned}b(e, x) &= \lambda_e e + \lambda_x x + \delta; \\P(r) &= \underbrace{\beta}_{ERC} r + \alpha.\end{aligned}$$

- Then, the manager solves:

$$\begin{aligned}\arg \max_b x [\beta (e + b) + \alpha] - \frac{1}{2}cb^2 \\ \implies b = \frac{\beta}{c}x.\end{aligned}$$

- Therefore, in any equilibrium, we must have $\lambda_e = 0$, $\delta = 0$, and $\lambda_x = \frac{\beta}{c}$.

Fischer and Verrecchia (2000)

- The market observes:

$$\tilde{r} = \tilde{e} + \frac{\beta}{c} \tilde{x} = \tilde{v} + \tilde{n} + \frac{\beta}{c} \tilde{x}$$

Two sources of noise: earnings noise and manipulation noise. We obtain:

$$E[\tilde{v}|\tilde{r}] = \frac{\sigma_v^2 \left(\tilde{r} - \frac{\beta}{c} \mu_x \right)}{\sigma_v^2 + \sigma_n^2 + \frac{\beta^2}{c^2} \sigma_x^2}.$$

- So, an equilibrium is defined by:

$$\begin{aligned} \beta &= \frac{\sigma_v^2}{\sigma_v^2 + \sigma_n^2 + \frac{\beta^2}{c^2} \sigma_x^2} \\ \Leftrightarrow \sigma_x^2 \beta^3 + (c^2 \sigma_n^2 + c^2 \sigma_v^2) \beta - c^2 \sigma_v^2 &= 0 \end{aligned}$$

- This equation has a unique solution $\beta^* \in \left(0, \frac{\sigma_v^2}{\sigma_v^2 + \sigma_n^2} \right)$.

What is special about $\frac{\sigma_v^2}{\sigma_v^2 + \sigma_n^2}$?

Fischer and Verrecchia (2000)

- Existence follows from the intermediate-value theorem:

$$\lim_{\beta \rightarrow 0} [\sigma_x^2 \beta^3 + (c^2 \sigma_n^2 + c^2 \sigma_v^2) \beta - c^2 \sigma_v^2] = -c^2 \sigma_v^2 < 0$$

$$\lim_{\beta \rightarrow \frac{\sigma_v^2}{\sigma_v^2 + \sigma_n^2}} [\sigma_x^2 \beta^3 + (c^2 \sigma_n^2 + c^2 \sigma_v^2) \beta - c^2 \sigma_v^2] = \frac{\sigma_v^6 \sigma_x^2}{(\sigma_n^2 + \sigma_v^2)^3} > 0.$$

- Uniqueness follows from the fact that:

$$\frac{\partial}{\partial \beta} [\sigma_x^2 \beta^3 + (c^2 \sigma_n^2 + c^2 \sigma_v^2) \beta - c^2 \sigma_v^2] = c^2 \sigma_n^2 + c^2 \sigma_v^2 + 3\beta^2 \sigma_x^2 > 0.$$

- We can conduct comparative statics on β by using the implicit function theorem.
- α captures the price correction for expected bias and equals $-\beta E[b(e, x)] = -\frac{\beta^2}{c} \mu_x$.

Fischer and Verrecchia (2000): Comparative Statics

- **Bias cost c and the ERC:**

$$\frac{\partial \beta}{\partial c} = \frac{-2\beta (\sigma_v^2 + \sigma_n^2) c + 2\sigma_v^2 c}{3\sigma_x^2 \beta^2 + c^2(\sigma_n^2 + \sigma_v^2)}.$$

Plugging in the equilibrium condition (a common trick) yields that this is positive. *Less costly bias reduces the market's response to earnings.*

- **Incentive uncertainty σ_x^2 and the ERC:**

$$\frac{\partial \beta}{\partial \sigma_x^2} = \frac{-\beta^3}{3\sigma_x^2 \beta^2 + c^2(\sigma_n^2 + \sigma_v^2)} < 0.$$

More uncertainty over the manager's incentives to manipulate reduces the market's response to earnings.

- σ_n^2, σ_v^2 impact the ERC in an intuitive manner.

Fischer and Verrecchia (2000): Comparative Statics

Unlike the model with fixed incentives, bias is not necessarily a deadweight loss to the manager.

- The manager's ex-ante expected utility is:

$$E \left[\tilde{x} (\beta (\tilde{e} + b) + \alpha) - \frac{cb^2}{2} \right] = \frac{\beta^2}{2c} (\sigma_x^2 - \mu_x^2).$$

On the other hand, if they do not bias, their expected utility is simply the expected price, or 0.

- So, they benefit from being able to bias whenever $\sigma_x^2 - \mu_x^2 > 0$.

Intuition:

- The manager biases more, thereby pushing up the price, precisely when they benefit more from a higher price, i.e., when \tilde{x} is higher.
- That is, managers with high short-term incentives have a negative externality on those with low short-term incentives.

- However, the managers with low short-term incentives don't care!

Without modeling where heterogeneity in incentives comes from, hard to say if bias has a social cost.

E.g., it might come out of investors' pockets in compensation, but contracts could adjust for this.

Fischer and Verrecchia (2000): Empirical Studies

Several recent empirical or joint theory/empirical papers have built on or directly tested Fischer and Verrecchia (2000).

- **Ferri, Zheng, Zou (2018 JAE):** compensation disclosure provides an incremental signal about \tilde{x} . F&V generates predictions on how this affects the coefficients in a regression of P on r .
- **Fang, Huang, and Wang (2018 JAR):** the potential for errors in reporting affects both σ_n^2 and lowers the cost to biasing c via a “camouflage” effect. Generates a non-monotonic relationship between misreporting and errors.
- **Samuels, Taylor, and Verrecchia (2021 JAE):** public scrutiny both lowers $\text{var}[\tilde{x}]$ and raises c . Generates non-monotonic relationship between scrutiny and misreporting.
- **Kim (2024 JAR):** information acquisition about either incentives \tilde{x} or cash flows \tilde{v} , captured by EDGAR downloads, and misreporting.

Dye and Sridhar (2004)

- Another version of the model has uncertainty baked into the cost function; manager solves

$$\arg \max_r P(r) - (r - e - \tilde{c})^2,$$

where \tilde{c} is normally distributed. You will solve this version in problem set 3.

- This alternative is more tractable and the equilibrium can be solved in closed form. However, certain comparative statics are different.
 - Bias does not interact with other elements of the model such as the ERC.

Stein (1989): Assumptions

- **Dynamic steady-state learning model.** “Natural” earnings follow a random walk with additional transitory noise:

$$\begin{aligned}\tilde{e}_t^n &= \tilde{z}_t + \overbrace{\tilde{v}_t}^{\text{transitory noise}} \\ \tilde{z}_t &= \tilde{z}_{t-1} + \tilde{u}_t \\ \tilde{u}_t &\sim N(0, \sigma_u^2) \\ \tilde{v}_t &\sim N(0, \sigma_v^2).\end{aligned}$$

- Note \tilde{z}_t is the persistent component of earnings, following a “random walk.”
- The market’s discount rate is r .

Stein (1989): Assumptions

- “Observed” earnings also depend upon an action taken by the firm’s manager b_t :

$$e_t = e_t^n + b_t - c(b_{t-1}).$$

- $c(b_{t-1})$ is the cost to this action.
 - The cost at **date t** affects **date $t + 1$** earnings. Myopia problem.
 - Assumption: this function is increasing and convex.
- The action b_t is related to real earnings management, for instance, the setting of price or intangible investment. b_t would have to capture *const.* – *intangible investment*
 - Paper also gives the example of “liquidating assets that are not ripe for liquidation.” But, emphasizes that to apply the results, it is crucial the behavior is unobservable.
 - Some problems in translating this to practice where intangible investment could be observable. Claim is that this practice is imperfect, e.g., some intangible investment may be misclassified as operating expenses.
- Unlike before, the efficient choice of b_t is not zero. The first best choice of b_t solves:

$$\frac{\partial}{\partial b_t} \left[b_t - \frac{c(b_t)}{1+r} \right] = 0 \implies c'(b_t^{FB}) = 1 + r.$$

Stein (1989): Earnings management

- Observed earnings in each period are immediately paid out as a dividend.

As a result: price is expected discounted future earnings:

$$P_t = E_t \left[\sum_{j=1}^{\infty} \frac{e_{t+j}}{(1+r)^j} \right]$$

- Managers enter each period owning x shares. They sell π fraction of these shares at the end of the period and hold the remainder indefinitely.
 - As we will see, their desire to sell creates a myopic incentive to increase b_t .
- Most easily understood as a new manager in each period. But, paper states that the results are not sensitive to the manager living multiple periods.
- Any rationale for the manager putting an excessive weight on current period stock price is sufficient. Could be career concerns, short-term compensation, or liquidation of shares.
- Formally, the manager chooses b_t to maximize:

$$U_t = x \times \left\{ \begin{array}{l} e_t \\ \text{dividend} \end{array} + \begin{array}{l} \pi P_t \\ \text{sale proceeds} \end{array} + (1 - \pi) \frac{e_{t+1}}{1+r} \right\}$$

residual value

Earnings more than one period into the future can be ignored, as b_t has no effect on those earnings.

Stein (1989): Equilibrium

- Suppose that the market conjectures a “steady-state” bias \bar{b} (i.e., bias is constant over time). Then, they can derive natural earnings from observed earnings:

$$\hat{e}_t^n = e_t + c(\bar{b}) - \bar{b}.$$

- From here, they can use results on the steady-state of the random walk plus transitory noise process to arrive at:

$$E_t(e_{t+k}^n) = \sum_{j=0}^{\infty} \alpha_j \hat{e}_{t-j}^n.$$

α_0 is the coefficient on today's earnings, α_1 is the coefficient on last period's earnings, and so on.

- **Notice:** the history of earnings continues to be relevant.
 - Past earnings are informative even given today's earnings because they help infer the persistent component of earnings z_t .
 - High past earnings indicate high z_t ; low past earnings indicate low z_t .
 - Since today's earnings are a noisy indicator of z_t , they are not a sufficient statistic for past earnings

Stein (1989): Equilibrium

$$E_t(e_{t+k}^n) = \sum_{j=0}^{\infty} \alpha_j \hat{e}_{t-j}^n.$$

- α_0 is of special interest – it captures the strength of the reaction of price to current earnings. It equals:

$$\alpha_0 = \underbrace{\sqrt{\frac{k^2}{4} + k} - \frac{k}{2}}_{\text{increases in } k} \quad \text{where } k = \frac{\sigma_u^2}{\sigma_v^2}.$$

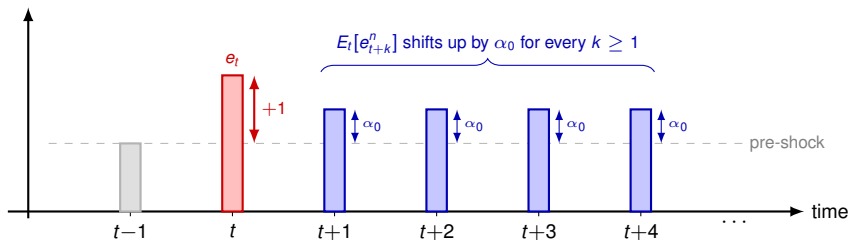
k captures how permanent vs. transitory earnings are.

Higher σ_v \implies more transitory earnings \implies lower price reaction.

Higher σ_u \implies more permanent earnings \implies greater price reaction.

Stein (1989): How e_t shifts expected future earnings

Increasing b_t boosts today's earnings \hat{e}_t^n , raising the market's posterior about the persistent component z_t , lifting *every* future expected earnings $E_t(e_{t+k}^n)$ by α_0 .



- Today's shock is partly transitory and partly persistent: only the fraction $\alpha_0 < 1$ survives into expectations of future earnings.
- Because the shift is the *same* α_0 at every future date, the price impact is a perpetuity:

$$\frac{dP_t}{de_t} = \sum_{j=1}^{\infty} \frac{\alpha_0}{(1+r)^j} = \frac{\alpha_0}{r}.$$

- The market does not adjust for the impact of future bias costs; these are fixed at its equilibrium conjecture $c(\bar{b})$.

Stein (1989): Manager's optimization

The manager's first-order condition yields:

$$\begin{aligned}0 &= \frac{dU_t}{db_t} = \frac{d}{db_t} \left[e_t + \pi P_t + (1 - \pi) \frac{e_{t+1}}{1 + r} \right] \\ \Leftrightarrow 0 &= \frac{de_t}{db_t} + \pi \frac{dP_t}{db_t} + \frac{1 - \pi}{1 + r} \frac{de_{t+1}}{db_t} \\ \Leftrightarrow 0 &= 1 + \pi \frac{\alpha_0}{r} - \frac{1 - \pi}{1 + r} c'(\bar{b}).\end{aligned}$$

This is an implicit solution for the manager's equilibrium action. Note in the quadratic cost case ($c(b) = \frac{\delta}{2} b^2$):

$$\begin{aligned}0 &= 1 + \pi \frac{\alpha_0}{r} - \frac{1 - \pi}{1 + r} \delta b \\ b &= \frac{(1 + r) \left(1 + \pi \frac{\alpha_0}{r} \right)}{\delta(1 - \pi)}\end{aligned}$$

Stein (1989): Comparative statics

$$b = \frac{(1+r) \left(1 + \pi \frac{\alpha_0}{r}\right)}{\delta(1-\pi)}$$

This yields:

1. $\frac{\partial b}{\partial \sigma_v^2} > 0$. Earnings of a more *permanent* nature increase the price response to contemporaneous earnings \implies more bias.
2. $\frac{\partial b}{\partial \sigma_v^2} < 0$. Earnings of a more *transitory* nature decrease the price response to contemporaneous earnings \implies less bias.
3. $\frac{\partial b}{\partial \pi} > 0$. Managers place a greater weight on price \implies more bias.