Disclosure and Financial Markets Lecture 7: Earnings Manipulation Part 2, Topics

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Agenda

1. Beyer, Guttman, and Marinovic (2019)
   Dynamic model of manipulation with uncertainty + structural estimation

2. Beyer (2009)
   Manipulation with uncertain risk

3. Fischer and Stocken (2004)
   Manipulation and informed trade

Beyer, Guttman, and Marinovic (2019): Overview

- Beyer, Guttman, and Marinovic (2019) analyze a dynamic model of earnings manipulation.
  - Their goal is to parse out earnings quality from economic uncertainty, both of which could influence existing discretionary accruals measures.
- Costly misreporting of *book value* (specifically, retained earnings) rather than earnings per se.
- Investors simultaneously update their beliefs about the firm's equity and future earnings.
- Investors face long-lasting uncertainty and asymmetry of information about B/S and I/S.
- Structurally estimate steady-state model.
  - Identification comes from variance and autocorrelation of earnings and prices.

Beyer, Guttman, and Marinovic (2019): Model

- A firm generates earnings, $\tilde{e}_t$, in each period $t = 1, 2, ...$

- Economic earnings follow an AR(1) process:

$$\tilde{e}_t = \rho \tilde{e}_{t-1} + \tilde{v}_t$$

  where $\tilde{v}_t \sim N(0, \sigma_v^2)$.

- There are no dividend distributions, hence pre-manipulation equity (retained earnings), $\tilde{\theta}_t$, is:

$$\tilde{\theta}_t = \underbrace{\tilde{\theta}_{t-1}}_{\text{last period's b.v.}} + \underbrace{\tilde{e}_t.}_{\text{earnings}}$$

- Investors observe reported book equity $\tilde{r}_t$ as opposed to true book equity $\tilde{\theta}_t$.

Beyer, Guttman, and Marinovic (2019): Model

- The manager may misreport the firm's equity, but misreporting is costly.

- Personal cost from reporting $\tilde{r}_t$ given true equity $\tilde{\theta}_t$ is:

$$\frac{c}{2}\left(\tilde{r}_t - \tilde{\theta}_t - \tilde{\eta}_t\right)^2,$$

  where $\tilde{\eta}_t$ is the shock to reporting incentives.
  Dye and Sridhar (2004) approach to modeling uncertainty over bias.

- $\tilde{\eta}_t$ follows an AR(1) process:

$$\tilde{\eta}_t = \phi\tilde{\eta}_{t-1} + \tilde{\varepsilon}_t,$$

  where $\tilde{\varepsilon}_t \sim N(0, \sigma_\eta^2)$. $\phi = 0$ in the main model; we'll focus on this. Rules out learning about incentives over time.

Beyer, Guttman, and Marinovic (2019): Model

In each period t, manager chooses $\tilde{r}_t$ to maximize:

$$U_t = \mathbb{E}\left[\sum_{k=t}^{T} \delta^{k-t}\left(p_k - \frac{c}{2}\left(r_k - \tilde{\theta}_k - \tilde{\eta}_k\right)^2\right)|\theta_t, e_t, \varepsilon_t\right].$$

Observe that:

1. The manager cares about prices and future bias costs in all future periods, but discounts the future at rate δ.
2. The manager conditions on book value and current earnings.
   From the manager's perspective: book value is a "state variable" that summarizes the relevant information in all past earnings realizations. (Follows because earnings are Markovian.)

Beyer, Guttman, and Marinovic (2019): Model

- In the absence of incurring additional manipulation costs in each period, reported book value (retained earnings) is tied to fundamental book value.
- Note while the manager reports book value in the model, we can alternatively think about them reporting earnings, as long as we interpret the cost appropriately.
- **One-to-one mapping:** given $\tilde{r}_{t-1}$, choosing a book value report $\tilde{r}_t$ is equivalent to choosing an earnings report $\tilde{r}^e_t \equiv \tilde{r}_t - \tilde{r}_{t-1}$, since $\tilde{r}_t = \tilde{r}_{t-1} + \tilde{r}^e_t$.
- Substitute into the cost, then use $\tilde{\theta}_t = \tilde{\theta}_{t-1} + \tilde{e}_t$ to bring earnings in:

$$
\begin{aligned}
\frac{c}{2}\left(\tilde{r}_t - \tilde{\theta}_t - \tilde{\eta}_t\right)^2 &= \frac{c}{2}\left(\tilde{r}_{t-1} + \tilde{r}^e_t - \tilde{\theta}_t - \tilde{\eta}_t\right)^2 \\
&= \frac{c}{2}\left(\tilde{r}^e_t - \tilde{e}_t + \underbrace{\left(\tilde{r}_{t-1} - \tilde{\theta}_{t-1}\right)}_{\equiv b_{t-1}} - \tilde{\eta}_t\right)^2 \\
&= \frac{c}{2}\left(\tilde{r}^e_t - \left(\tilde{e}_t - b_{t-1}\right) - \tilde{\eta}_t\right)^2.
\end{aligned}
$$

Beyer, Guttman, and Marinovic (2019): Model

- Recall, after substitution, the cost in earnings-report space is

$$\frac{c}{2}\left(\tilde{r}_t^e - \left(\tilde{e}_t - b_{t-1}\right) - \tilde{\eta}_t\right)^2, \quad b_{t-1} \equiv \tilde{r}_{t-1} - \tilde{\theta}_{t-1}.$$

- The zero-cost target (absent $\tilde{\eta}_t$) is actual earnings *minus* last period's bias: $\tilde{e}_t - b_{t-1}$.

- If the manager inflated book value last period ($b_{t-1} > 0$), they must under-report earnings this period to keep costs low; if they deflated it, they must over-report.

- Hence, the model captures accrual-based manipulation in which today's manipulation mechanically reverses tomorrow.
  E.g., booking next year's revenue in December: book value is high this period, but next period's reported earnings must be lower to keep the report close to fundamentals at zero cost.

Beyer, Guttman, and Marinovic (2019): Model

Risk-neutral pricing. Assuming the interest rate is zero, period t price is given by:

$$p_t = \mathbb{E}\left[\tilde{\theta}_t + \sum_{k=t+1}^{\infty} \tilde{e}_k | \boldsymbol{r}_t\right] = \overbrace{\mathbb{E}\left[\tilde{\theta}_t + \sum_{k=1}^{\infty} \left(\rho^k \tilde{e}_t + noise\right) | \boldsymbol{r}_t\right]}^{\text{AR(1) earnings structure}}$$

$$= \mathbb{E}\left[\tilde{\theta}_t + \tilde{e}_t \sum_{k=1}^{\infty} \rho^k | \boldsymbol{r}_t\right]$$

$$= \mathbb{E}_t\left[\tilde{z}_t\right]$$

where:

$$\tilde{z}_t \equiv \tilde{\theta}_t + \frac{\rho}{1-\rho}\tilde{e}_t$$

is the firm's economic/intrinsic/fundamental value.

This follows from the fact that $\sum_{k=1}^{\infty} \rho^k = \frac{\rho}{1-\rho}$.

Beyer, Guttman, and Marinovic (2019): Finite Period Case

Consider first a manager who runs the firm until date $T < \infty$.
There exists a unique linear equilibrium characterized as follows:

- The date t price satisfies:

$$p_t = \mu_0 + \sum_{j=1}^{t} \alpha_j^t \left(\tilde{r}_j - b_j\right),$$

  where $\left\{\alpha_j^t, b_j\right\}_{j=1}^{t}$ are constants that depend on $\left\{\rho, \phi, \sigma_v, \sigma_\eta\right\}$.
- Note earnings impact future prices as in Stein (1989). Occurs because incentives are i.i.d., so past signals continue to be relevant into the future.
- Let δ be the manager's discount rate. Their reporting strategy is:

$$\tilde{r}_t = \tilde{\theta}_t + \underbrace{\tilde{\eta}_t + A_t^T}_{\text{bias}} \text{ where } A_t^T = \frac{\sum_{k=0}^{T-t} \delta^k \alpha_t^{t+k}}{c}.$$

Beyer, Guttman, and Marinovic (2019): Finite Period Case

Note from the firm's perspective, expected firm value at date t is

$$\tilde{z}_t = \mathbb{E}[\text{Firm Value}|\tilde{e}_t] = \mathbb{E}\left[\tilde{\theta}_t + \sum_{k=t+1}^{T} \tilde{e}_k | \tilde{e}_t\right] = \tilde{\theta}_t + \tilde{e}_t \sum_{k=1}^{T-t} \rho^k.$$

Together with $\tilde{r}_t = \tilde{\theta}_t + \tilde{\eta}_t + A_t^T$, this yields

$$\tilde{r}_t = \tilde{z}_t \underbrace{-\tilde{e}_t \sum_{k=1}^{T-t} \rho^k + \tilde{\eta}_t + A_t^T}_{\substack{\text{"error" in the report relative} \\ \text{to unbiased fair value report}}}.$$

- The error in the report as a metric of firm value is negatively correlated with the level of firm value given the auto-regressive nature of earnings.
- Captures the conservative nature of book equity.

Beyer, Guttman, and Marinovic (2019): Earnings Quality

The paper defines earnings quality EQ_t as:

$$EQ_t \equiv -Var_t\left(\tilde{z}_t\right) < 0,$$

where:

- $\tilde{z}_t$ is the firm's economic value given the information available to the firm at time t;
- Var_t is the variance conditional on the information available to investors $\{r_1, r_2, .., r_t\}$.

Hence, EQ_t measures the information asymmetry between the manager and investors.

In the absence of misreporting, or if the manager's bias cost was known, their information sets would be identical, and so $EQ_t = 0$.

Beyer, Guttman, and Marinovic (2019): Earnings Quality

- EQ_t decreases in economic uncertainty $\{\sigma_v, \rho\}$.
  - Scales up the overall amount of uncertainty over firm value.
- EQ_t decreases in uncertainty over the manager's bias cost σ_η.
- EQ_t is independent of the manager's discount factor δ and the cost of misreporting c.
  - Related to results we saw last class: this would change in a Fischer and Verrecchia setting.
  - Aggressive manipulation can be consistent with high EQ, if it is anticipated and corrected for by investors. This could be a problem for discretionary accruals metrics.

Beyer, Guttman, and Marinovic (2019): Pricing

The price expression can be rewritten as follows:

$$p_t = p_{t-1} + \beta_t \left(\tilde{r}_t - \mathbb{E}_{t-1}\left(\tilde{r}_t\right)\right) \tag{1}$$

where $\beta_t = \frac{Cov_{t-1}(\tilde{r}_t, \tilde{z}_t)}{Var_{t-1}(\tilde{r}_t)}$.

- This shows that the price is a martingale (always holds in risk-neutral settings, after discounting).
- They next analyze a steady state: represents the case when t and $T - t$ are both large.
- In steady state, investors' learning process is stable. Price function becomes:

$$p_t = p_{t-1} + \beta(\tilde{r}_t - \mathbb{E}_{t-1}(\tilde{r}_t)). \tag{2}$$

- Solve for a steady state by conjecturing and verifying or, in some cases, by analyzing the limit of the finite-time case.

Beyer, Guttman, and Marinovic (2019): Steady State

- Let:

$$\Sigma = \begin{pmatrix} \sigma_\theta^2 & \sigma_{\theta e} & \sigma_{\theta\eta} \\ . & \sigma_e^2 & \sigma_{e\eta} \\ . & . & \sigma_\eta^2 \end{pmatrix}$$

  be the steady-state variance covariance matrix of $\{\theta, e, \eta\}$.
  Steady state $\Leftrightarrow$ Σ is independent of t.

- Σ is a function of the parameters that determine the information structure $\{\rho, \phi, \sigma_v, \sigma_\eta\}$.

Beyer, Guttman, and Marinovic (2019): Steady State

In steady state, investors update Σ using Bayes' rule for MV normals:

$$\underbrace{\begin{bmatrix} Var_{t-1}(\theta_t) & Cov_{t-1}(\theta_t, e_t) & Cov_{t-1}(\theta_t, \eta_t) \\ Cov_{t-1}(\theta_t, e_t) & Var_{t-1}(e_t) & Cov_{t-1}(e_t, \eta_t) \\ Cov_{t-1}(\theta_t, \eta_t) & Cov_{t-1}(e_t, \eta_t) & Var_{t-1}(\eta_t) \end{bmatrix}}_{\text{prior uncertainty}}$$

$$-\underbrace{\frac{\begin{bmatrix} Cov_{t-1}(\theta_t, r_t) \\ Cov_{t-1}(e_t, r_t) \\ Cov_{t-1}(\eta_t, r_t) \end{bmatrix} \begin{bmatrix} Cov_{t-1}(\theta_t, r_t) \\ Cov_{t-1}(e_t, r_t) \\ Cov_{t-1}(\eta_t, r_t) \end{bmatrix}'}{Var_{t-1}(r_t)}}_{\text{information}}$$

$$= \underbrace{\begin{bmatrix} Var_t(\theta_t) & Cov_t(\theta_t, e_t) & Cov_t(\theta_t, \eta_t) \\ Cov_t(\theta_t, e_t) & Var_t(e_t) & Cov_t(e_t, \eta_t) \\ Cov_t(\theta_t, \eta_t) & Cov_t(e_t, \eta_t) & Var_t(\eta_t) \end{bmatrix}}_{\text{posterior uncertainty } (\Sigma)}$$

Beyer, Guttman, and Marinovic (2019): Steady State

- The equations for steady state lead to an implicit solution for Σ.
- The solution is complex. The paper turns to structurally estimating the model.
- **Key intermediate step:** upon performing manipulations, BGM show the following, where $\gamma = \frac{d\mathbb{E}[e_t]}{dr_t}$ is an endogenous function of the model's parameters.

Corollary

In the steady state, the price change is characterized by the difference equation:

$$\Delta p_t = \rho\left(1 + \gamma\frac{\rho}{1-\rho}\right)\Delta p_{t-1} + \beta\left(r_t - p_{t-1}\right) - \rho\beta\left(r_{t-1} - p_{t-2}\right).$$

Beyer, Guttman, and Marinovic (2019): Estimation

- Price data does not match the deterministic model of reports and past prices, because prices are noisy.

- To capture this, before bringing the model to the data, they must allow for noise somewhere in the model.

- **Solution:** suppose we observe $\tilde{p}_t = p_t + \tilde{\zeta}_t$ where $\tilde{\zeta}_t$ is white noise.
  Econometric model:

$$\Delta\tilde{p}_t = \rho\left(1 + \gamma\frac{\rho}{1-\rho}\right)\Delta\tilde{p}_{t-1} + \beta\left(r_t - \tilde{p}_{t-1}\right) - \rho\beta\left(r_{t-1} - \tilde{p}_{t-2}\right) + \tilde{n}_t,$$

  where:

$$\begin{aligned}\tilde{n}_t &= \tilde{\zeta}_t - \tilde{\zeta}_{t-1} - \rho\left(1 + \gamma\frac{\rho}{1-\rho}\right)\left(\tilde{\zeta}_{t-1} - \tilde{\zeta}_{t-2}\right) \\ &\quad + \beta\tilde{\zeta}_{t-1} - \rho\beta\tilde{\zeta}_{t-2}.\end{aligned}$$

- Note $\tilde{n}_t$ is correlated with explanatory variables. Hence, OLS doesn't work as an estimation procedure.

Beyer, Guttman, and Marinovic (2019): Estimation

Common structural methods:

1. **Maximum likelihood estimation**: find the parameters that maximize the likelihood of observing the data.
   - Fully "parametric:" requires specifying the underlying distributions.
2. **Method of moments**: find the parameters that equate the theoretical moments to those in the data.
   - Moments not selected can be used to perform a goodness-of-fit test.
   - Less "parametric" than MLE. E.g., asset-pricing models often deliver expected return relationships that hold regardless of distribution.
   - Less powerful than MLE if one truly knows the distribution, but can be more robust if one does not.
3. **Generalized method of moments**: find the parameters that minimize the difference between a weighted average of a selection of theoretical moments and the observed moments.

Beyer, Guttman, and Marinovic (2019): Estimation

They apply a two-step procedure that combines MLE+MoM:

1. Estimate $\{\beta, \gamma, \rho\}$ from the price equation via MLE.
   Joint dynamics of prices and reports reveal how strongly investors revise their beliefs about true earnings from reported earnings and how persistent earnings are.
2. Based on the estimates, $\left\{\hat{\beta}, \hat{\gamma}, \hat{\rho}\right\}$, use the steady-state conditions for β and γ to obtain estimates for $\{\sigma_v, \sigma_\eta\}$.

Step 2 involves some work.

The price-change coefficients $\left\{\hat{\beta}, \hat{\gamma}, \hat{\rho}\right\}$ do not immediately identify the values of $\{\sigma_v, \sigma_\eta\}$ because they depend only on the ratio $\frac{\sigma_v}{\sigma_\eta}$, not their levels.
This is a common theme in structural work. One must work to find elements of the data to separately estimate certain parameters.

Beyer, Guttman, and Marinovic (2019): Estimation

Define:

$$m_t \equiv \Delta r_t - \rho \Delta r_{t-1}.$$

Use MoM to estimate $\{\sigma_v, \sigma_\eta\}$ given MLE estimate for ρ:

$$\begin{aligned} \widehat{Var(m_t)} - \left[\sigma_v^2 + 2\left(1+\rho+\rho^2\right)\sigma_\eta^2\right] &= 0; \\ Cov\widehat{(m_t, m}_{t-1}) + (1-\rho)^2 \sigma_\eta^2 &= 0. \end{aligned}$$

Rough idea: the two parameters can be separated from the time series of reported earnings because incentives are i.i.d. while earnings are persistent.

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Beyer (2009): Background

- Beyer (2009) brings us back to the topic of learning about risk from earnings.

- In this paper, a manager issues an earnings forecast that they can bias in advance of earnings. Critically, investors are uncertain about the variance of firm value.

- The paper studies the optimal choice of forecast and earnings biases when the forecast error is used to assess the cash-flow variance.
  Contrasts with Beyer and Smith (2022), who looked at static learning about covariance/betas from earnings and macro-aggregates.

- **Relevant empirical questions:**
  1. Why does the market respond not only to earnings and earnings forecasts, but also to the forecast *error*?
  2. Why do managers issue pessimistic forecasts that they are likely to beat?
  3. Why do investors weight earnings more heavily than earnings forecasts?

Beyer (2009): Background

Important assumptions:

- The manager has imperfect information at the time of the forecast.
  $\Longrightarrow$ absolute forecast error and risk are correlated.
- Manager has no leeway regarding whether to issue a forecast.
  Contrasts with voluntary disclosure models of earnings forecasts; we'll return to this next class.

To start: let's look at learning about variance from multiple signals.

- For tractable Bayesian updating, must consider a new distribution.
  Normal-Inverse-Gamma.

Beyer (2009): Bayes' Rule for Normal-Inverse-Gamma

Theorem

Suppose $\tilde{x}|\tilde{\tau} \sim N\left(\mu, k\tilde{\tau}^{-1}\right)$ *where* $\tilde{\tau} \sim Gamma\left(\alpha, \beta\right)$. *Suppose further that one observes n signals:*

$$\tilde{s}_i = \tilde{x} + \tilde{\varepsilon}_i,$$

where $\tilde{\varepsilon}_i|\tilde{\tau} \sim N\left(0, \tilde{\tau}^{-1}\right)$ *is independent of* $\tilde{x}$ *and* $\tilde{\varepsilon}_j$ *given* $\tilde{\tau}$. *Then:*

$$\begin{aligned}
\tilde{x}|\left\{\tilde{s}_i\right\}, \tilde{\tau} \quad &\sim \quad N\left(\frac{k\mu + \sum \tilde{s}_i}{k+n}, \frac{1}{\tilde{\tau}}\frac{1}{k+n}\right); \\
\tilde{\tau}|\left\{\tilde{s}_i\right\} \quad &\sim \quad Gamma\left(\alpha + \frac{n}{2}, \beta + \frac{1}{2}\sum\left(\tilde{s}_i - \frac{\sum \tilde{s}_i}{n}\right)^2\right. \\
&\qquad \left. + \left(\frac{nk}{n+k}\right)\frac{\left(\frac{\sum \tilde{s}_i}{n} - \mu\right)^2}{2}\right).
\end{aligned}$$

- Note $\tilde{\tau}^{-1}$ has an inverse-gamma distribution.

Beyer (2009): Bayes' Rule for Normal-Inverse-Gamma

By assumption, the precisions of the error term $\tilde{\varepsilon}_j$ and the underlying cash flow $\tilde{x}$ are proportional.

- This is necessary for (1) a linear posterior mean (2) a closed-form solution for the posterior.
- Intuition: otherwise, a more extreme signal would suggest that the signal was more or less precise relative to the prior.
- As a result, more extreme signals would effectively get less weight, leading to a non-linear response to the signal (remember Subramanyam (1995)).

Beyer (2009): Bayes' Rule for Normal-Inverse-Gamma

$$
\begin{aligned}
Var\left[\tilde{x} \middle| \{\tilde{s}_i\}\right] &= \frac{E\left[\tilde{\tau}^{-1} \middle| \{\tilde{s}_i\}\right]}{k+n} \qquad \text{(law of total variance)} \\
&= \frac{1}{k+n} \frac{\beta^{Post}}{\alpha^{Post}-1} \\
&= \frac{\beta + \frac{1}{2}\sum\left(\tilde{s}_i - \frac{\sum \tilde{s}_i}{n}\right)^2 + \frac{1}{2}\frac{nk}{n+k}\left(\frac{\sum \tilde{s}_i}{n} - \mu\right)^2}{(k+n)\left(\alpha + \frac{n}{2} - 1\right)}.
\end{aligned}
$$

Two features of the data induce a less risky posterior distribution:

1. Consistent signals: low $\sum\left(\tilde{s}_i - \frac{\sum \tilde{s}_i}{n}\right)^2$.
2. Signals closer to prior mean: low $\left(\frac{\sum \tilde{s}_i}{n} - \mu\right)^2$.

There is some empirical evidence that more extreme earnings realizations increase perceptions of risk, consistent with the theorem (Neururer et al. (2016 RAST)).

- Implied vol. goes up following earnings with more extreme realizations.
- Smith and So (2021) also find this result using their "corrected" measure.

Beyer (2009): Bayes' Rule for Normal-Inverse-Gamma

- Beyer (2009) will focus on the case with **one signal (earnings)** that is used to update on the variance, in which case the relevant formula reduces to**:**

$$\tilde{x}|\tilde{s},\tilde{\tau} \sim N\left(\frac{k\mu+\tilde{s}}{k+1}, \frac{\tilde{\tau}^{-1}}{k+1}\right);$$
$$\tilde{\tau}|\tilde{s} \sim Gamma\left(\alpha+\frac{1}{2}, \beta+\frac{k}{1+k}\frac{(\tilde{s}-\mu)^2}{2}\right).$$

- Earnings closer to the prior mean will lead investors to perceive lower risk.

- Investors will also use the forecast in assessing risk because it will affect the prior mean μ.

Beyer (2009): Model Set Up

- The firm has two components of cash flow, $\tilde{x}_1, \tilde{x}_2$. The information structure is:

$$\tilde{x}_1 = \tilde{\mu}_1 + \tilde{v}_1; \qquad \tilde{x}_2 = \tilde{\mu}_2 + \tilde{v}_2;$$
$$Var[\tilde{v}_1|\tilde{\tau}] = Var[\tilde{v}_2|\tilde{\tau}] = \tilde{\tau}^{-1};$$
$$Var[\tilde{\mu}_1] = Var[\tilde{\mu}_2] = \sigma_u^2;$$
$$Corr[\tilde{\mu}_1, \tilde{\mu}_2] = \rho.$$

  No agent has information on $\tilde{x}_2$; its role is to ensure that uncertainty over the variance continues to play a role even after date 1 cash flows are known.

- First, the manager issues a forecast after observing $\tilde{\mu}_1$.
  - Allows the manager to make an (imperfect) forecast regarding $\tilde{x}_1$.

- Second, she issues earnings after observing $\tilde{x}_1$. **Her goal is to maximize price after the market observes both the forecast and earnings**.
  - That is, she does not care about the price between forecast and earnings.
  - Firm is priced by mean-variance investors:
    $P = E[\tilde{x}_1 + \tilde{x}_2|\Omega] - \gamma Var[\tilde{x}_1 + \tilde{x}_2|\Omega]$.

Beyer (2009): Technical Aside

- Mean-variance pricing is important for tractability.

- CARA utility would not work as the cash-flow distribution is non-normal.
  - In fact, the normal-inverse-gamma does not have a moment-generating function.
  - Technically, no utility function generates preferences that are linear in mean and variance for all distributions.

- Yet, this is not a major concern in my view.
  - The results have a compelling and general intuition that would seem to generalize to other utility functions.

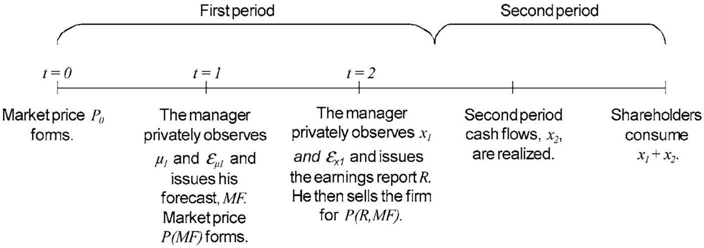
Beyer (2009): Model Set Up

Manager can bias both the forecast and the earnings number. She has a random component in her bias costs:

- Cost to bias forecast $\propto (bias - \tilde{\varepsilon}_{\mu 1})^2$ where $Var[\tilde{\varepsilon}_{\mu 1}] = k_1 \sigma_u^2$;
- Cost to bias earnings $\propto (bias - \tilde{\varepsilon}_{\mu 1} - \tilde{\varepsilon}_{v1})^2$ where $Var[\tilde{\varepsilon}_{v1}] = k_2 \tilde{\tau}^{-1}$.

“This attempts to capture that, in practice, capital markets may not have the ability to make perfect inferences about a manager’s private information regarding the performance of a company.”

FIGURE 1
Sequence of Events



Beyer (2009): Equilibrium

- The manager's forecast bias choice is similar to Dye and Sridhar:

$$MF = \tilde{\mu}_1 + \underbrace{\tilde{\varepsilon}_{\mu 1} + m_0}_{\text{manager's bias}}.$$

- In contrast, earnings bias is conceptually distinct from the problems we have studied thus far.

- The bias depends upon both the forecast and $\tilde{x}_1$:

$$R = r_0 + r_1 MF + r_2 \left(\tilde{x}_1 + \tilde{\varepsilon}_{\mu 1} + \tilde{\varepsilon}_{v1}\right).$$

- The forecast is relevant to bias choice as the forecast error influences investors' perception of the firm's risk.

Beyer (2009): Equilibrium

The market can de-bias the forecast and earnings to arrive at "truth-plus-noise" signals:

$$
\begin{aligned}
\tilde{y}_{\mu 1} &\equiv MF - m_0 = \tilde{\mu}_1 + \tilde{\varepsilon}_{\mu 1}; \\
\tilde{y}_{x1} &\equiv \frac{R - r_0 - r_1 MF}{r_2} = \tilde{x}_1 + \tilde{\varepsilon}_{\mu 1} + \tilde{\varepsilon}_{v1}.
\end{aligned}
$$

1. The adjusted forecast lets investors update on $\tilde{\mu}_1$. This follows standard normal-normal updating.
2. Earnings allow investors to update on $\tilde{x}_1$. This can be calculated using the earlier formula for the normal-inverse-gamma.

Importantly, the conditional variance has a quadratic term:

$$
\frac{(\tilde{y}_{x1} - \tilde{y}_{\mu 1})^2}{2}.
$$

A greater deviation between the adjusted forecast and adjusted earnings is indicative of a higher variance. This term enters the equilibrium price.

Beyer (2009): Results in Invertible Bias Case

Suppose first that there is no uncertainty over bias, i.e., $\tilde{\varepsilon}_{\mu 1}$ and $\tilde{\varepsilon}_{v1}$ go away.

- Investors perfectly back out the manager's bias and:

$$P(R, MF) = \pi_0 + \pi_1 MF + \pi_2 R + \pi_3 (\text{Forecast error})^2$$

  where $\pi_1, \pi_3 < 0$ and $\pi_0, \pi_2 > 0$.

- Since $\pi_2 > 0$, the manager, on average, biases *earnings* upwards. And, since $\pi_3 < 0$, they bias to reduce the forecast error.
  - If earnings are surprisingly high to the manager, they reduce their bias to lower forecast error.
  - This reduces the sensitivity of earnings to cash flows, i.e., it reduces $r_2 = \frac{\partial R}{\partial x_1}$.
  - The ERC rises to compensate for this! Intuitively, recall the market divides R by r_2 when de-biasing the forecast, so a lower r_2 equates to a stronger market response.

Beyer (2009): Results in Invertible Bias Case

"When managers adhere to this equilibrium reporting strategy, the capital market values firms with positive earnings surprises at a premium. That is, controlling for the level of reported earnings and the magnitude of the earnings surprise, firms with earnings exceeding their forecast will enjoy a higher capital market valuation than firms that disappoint investors by reporting earnings below their forecast. (pg. 1714)"

- **Rationale**: Firms with earnings slightly above the forecast have lower risk than those slightly below.
- Intuitively, as the forecast is biased down and earnings up, the forecast error is, on average, positive.
- So, starting from zero, an increase in forecast error brings this error closer to the prior mean, reducing perceptions of variance.

"Fourth, the model predicts that the sensitivity of a firm's stock price to the earnings forecast is smaller than to the actual earnings announcement and that the difference between those two response coefficients varies systematically with the persistence of cash flows. (pg. 1714)"

- **Rationale**: stems from the smoothing effect whereby the market must amplify the deviation between earnings and the forecast to get to true earnings.

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Fischer and Stocken (2004): Background

- This paper studies how informed trade affects earnings manipulation and the resultant effect on earnings quality. Two-way interaction:
  - Informed traders see through manipulation and trade against it. This affects price efficiency.
  - The manager's incentives to manipulate may depend on price efficiency.

- Related to McNichols and Trueman. Key difference: earnings noise endogenously stems from uncertainty over the manager's manipulation incentives.

- The informed trader has info. on both cash flows and on manipulation incentives.
  - Their results revolve around the ERC, which drives the amount of manipulation.
  - If there is more informed trade on the fundamental, then the ERC goes down
    $\Rightarrow$ manipulation goes down. *Substitutive effect.*
  - If there is more informed trade on incentives, then the ERC goes up
    $\Rightarrow$ manipulation goes up. *Complementary effect.*

Fischer and Stocken (2004): Assumptions

- Three periods: pre- and post-earnings trade. Each round of trade is a Kyle model.
- Firm's value is the sum of two independent components: $\tilde{\varepsilon} = \tilde{\varepsilon}_1 + \tilde{\varepsilon}_2$.
- The manager issues a report e that can be biased. Similar to Fischer and Verrecchia, the manager solves:

$$e\left(\tilde{\varepsilon}, \tilde{m}\right) = \arg\max_{e(\tilde{\varepsilon}, \tilde{m})} \tilde{m}\tilde{P}_2 - \frac{\left(e - \tilde{\varepsilon}\right)^2}{2}$$

  where $\tilde{m} = \tilde{m}_1 + \tilde{m}_2$.

- A speculator, before trading in period 1, privately observes $\tilde{\varepsilon}_1$ and $\tilde{m}_1$.
  - They have a **short-term horizon** as in McNichols and Trueman.
- Liquidity trade in both periods $\tilde{l}_1, \tilde{l}_2 \sim N\left(0, \sigma_l^2\right)$.
  - Unlike McNichols and Trueman, these are independent.

Fischer and Stocken (2004): Assumptions

- The sub-components of cash flows and manipulation incentives have distributions:

$$\begin{aligned} \tilde{m}_1 &\sim N\left(\mu_{m1}, q_m \sigma_m^2\right) && \text{observed by informed trader} \\ \tilde{m}_2 &\sim N\left(\mu_{m2}, (1-q_m)\,\sigma_m^2\right) \\ \tilde{\varepsilon}_1 &\sim N\left(0, q_\varepsilon \sigma_\varepsilon^2\right) && \text{observed by informed trader} \\ \tilde{\varepsilon}_2 &\sim N\left(0, (1-q_\varepsilon)\,\sigma_\varepsilon^2\right). \end{aligned}$$

  where $q_m, q_\varepsilon \in [0, 1)$.

- q_m and q_ε control the speculator's information advantage regarding fundamentals and manipulation, respectively.

Fischer and Stocken (2004): Equilibrium

- They focus on linear equilibria:

$$\begin{aligned}\tilde{e} &= e_0 + e_\varepsilon \tilde{\varepsilon} + e_m \tilde{m}; \\ \tilde{d}_s &= d_0 + d_\varepsilon \tilde{\varepsilon}_1 + d_m \tilde{m}_1; \\ P_1 &= p_1^0 + p_1^{d_1} \tilde{d}_1; \\ P_2 &= p_2^0 + p_2^{d_1} \tilde{d}_1 + p_2^{d_2} \tilde{d}_2 + p_2^e \tilde{e}.\end{aligned}$$

- Compared to Fischer and Verrecchia:
  - The conjecture now concerns earnings rather than bias.
  - However, the approaches are equivalent: in equilibrium, we have $e_\varepsilon = 1$, $e_0 = 0$ and $e_m = p_2^e$. So, bias does not depend upon earnings, there is no bias when $\tilde{m} = 0$, and bias is proportional to the ERC.
- The equilibrium derivation is burdensome. If you want to test/practice your abilities and have a large amount of free time, see if you can figure it out.

Fischer and Stocken (2004): Equilibrium

PROPOSITION 1. *There exists a unique linear equilibrium with the characteristics:*

1. The speculator's demand order increases in information about the firm's fundamental earnings, $\tilde{\varepsilon}_1$ and the manager's reporting incentives, $\tilde{m}_1$;
2. the manager's earnings report increases in the realization of both fundamental earnings, $\tilde{\varepsilon}$, and incentive parameter, $\tilde{m}$;
3. the first-period price the market maker posts increases in the net demand order, $\tilde{d}_1$; and
4. the second-period price the market maker posts increases in the manager's earnings report, $\tilde{e}$, increases or decreases in the net demand order submitted in the first period, $\tilde{d}_1$, and decreases or increases in the net demand order submitted in the second period, $\tilde{d}_2$.

Fischer and Stocken (2004): Earnings Quality

$$\begin{aligned} EQ &\equiv E\left[-\left(\tilde{e}-\tilde{\varepsilon}\right)^2\right] \\ &= -e_m^2\left(\sigma_m^2+\mu_m^2\right). \end{aligned}$$

Note this differs from BGM's definition from earlier.

- EQ falls in μ_m, despite the expected bias being invertible. This metric is closer to discretionary accruals measures, which have the same feature.
- BGM's variance metric gets rid of the invertible component by analyzing the conditional variance. Thus, it is closer to the theoretical construct of interest.

Fischer and Stocken (2004): Earnings Quality

Let $\tilde{e}_n$ denote earnings without a speculator.

PROPOSITION 2. *A speculator causes earnings quality to decline if and only if the speculator's share of information regarding the managed component of earnings exceeds that regarding the fundamental component of earnings: that is,* $E\left[-\left(\tilde{e}-\tilde{\varepsilon}\right)^2\right] < E\left[-\left(\tilde{e}_n-\tilde{\varepsilon}\right)^2\right]$ *if and only if* $q_m > q_\varepsilon$.

Viewing the speculator as an analyst and f as her earnings forecast $E\left[\tilde{e}|\varepsilon_1, m_1\right]$, they argue $q_m - q_\varepsilon$ can be measured using a regression:

$$P = \beta_0 + \beta_f \tilde{f} + \beta_e \tilde{e} + \tilde{n}.$$

They show that $\beta_f > 0 \Rightarrow q_\varepsilon > q_m$ and $\beta_f < 0 \Rightarrow q_m > q_\varepsilon$.

Intuition: if price reacts positively to the forecast conditional on earnings, the forecast must be a better metric of true cash flows rather than anticipated manipulation.

Fischer and Stocken (2004): Price Efficiency

F&S next examine residual uncertainty in the presence of a speculator versus in the absence of a speculator, $Var\left(\tilde{\varepsilon}|\tilde{e},\tilde{d}_1,\tilde{d}_2\right)$ and $Var\left(\tilde{\varepsilon}|\tilde{e}_n\right)$.

PROPOSITION 3. *The speculator's presence increases price efficiency if the speculator's share of information about the fundamental component of earnings exceeds that about the managed components of earnings. The speculator's presence can decrease price efficiency if the speculator's share of information about the managed component of earnings exceeds that about the fundamental component of earnings; more specifically, for any* $q_\varepsilon > 0$*, there exists a threshold* $\bar{q}_m > q_\varepsilon$ *such that* $Var\left(\tilde{\varepsilon}|\tilde{e},\tilde{d}_1,\tilde{d}_2\right) > Var\left(\tilde{\varepsilon}|\tilde{e}_n\right)$ *if and only if* $q_m \in (q_\varepsilon, \bar{q}_m]$ *where* $\bar{q}_m \leq 1$*.*

Intuition: the more the speculator knows about manipulation, the greater the ERC $\Rightarrow$ the more manipulation that occurs in equilibrium.

For certain parameters, this can outweigh the direct inference made from the speculator's demand.