Disclosure and Financial Markets Lecture 8: Voluntary Disclosure; Product Markets

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Agenda

- Unravelling
- Verrecchia (1983)
  Costly disclosure
- Dye (1985)/Jung and Kwon (1988)
  Probabilistic information endowment
- Product Markets
  Endogenizing proprietary costs

(*) These notes are based in part upon Jeremy Bertomeu and Edwige Cheynel's excellent slides on voluntary disclosure.

Voluntary Disclosure: Overview

- In the models we have considered so far, the firm always releases a signal.
- Voluntary disclosure models: the firm has the option to disclose or not disclose a signal after observing its value.
- If they choose to disclose the signal, it is revealed truthfully (hard information).
  - For example, an audit/certifier or the potential for litigation may ensure the signal is truthful if disclosed.
- When does the firm disclose its signal?
- Mathematical results tend to be more analytical and less algebraic.

Voluntary Disclosure: Overview

- These models aren't a perfect fit to an individual setting, but loosely speak to numerous settings.
  - Management forecasts.
  - Project outcomes, e.g., pharmaceutical trials.
  - Conference calls.
  - Social media disclosures.

- Outside of a finance context:
  - Grade disclosure.
  - President's tax form disclosure.
  - Pleading the 5th.
  - Many more...

- The key insight from economics concerns the inference that can be made from non-disclosure and how this feeds back into the disclosure decision.

Voluntary Disclosure: Unravelling

- Firm privately observes future cash flow $\tilde{x}$, with mean μ, continuously distributed on $[a, b]$.
- Makes a **"verifiable"** disclosure decision: $D(\tilde{x}) \in \{\tilde{x}, ND\}$.
- Investors are risk neutral such that $P(D) = E[\tilde{x}|D(\tilde{x}) = D]$.
  - It need not be a price; we can think about this as an arbitrary action in the more general case.

Definition

An equilibrium is a price function $P(\cdot)$ mapping the observed disclosure D to a price, and a disclosure strategy $D(\tilde{x})$ such that:
(a) $P(D) = \mathbb{E}[\tilde{x}|D(\tilde{x}) = D]$ on the equilibrium path;
(b) $D(\tilde{x}) = \tilde{x}$ if and only if $\tilde{x} \geq P(ND)$.

- We will be dealing with expectations of r.v.'s truncated from above in these models, i.e., expressions of the form $\mathbb{E}(\tilde{x}|\tilde{x} \leq T)$.
  - Critically, these expectations increase in T (should be fairly obvious).

Voluntary Disclosure: Unravelling

Theorem

There exists a unique equilibrium in which the firm always discloses.

Intuition:

- Suppose students have GPAs uniformly distributed on $[2.0, 4.0]$.
- Now, suppose only types $[3.8, 4]$ disclose. Then, non-disclosure leads to an expected GPA of 2.9. So, types $[2.9, 3.8]$ prefer to deviate to disclosing.
- But, if types $[2.9, 4]$ disclose, then non-disclosure leads to an expected GPA of 2.45. So, types $[2.45, 2.9]$ prefer to deviate to disclosing.
- And so on and so forth.

Voluntary Disclosure: Unravelling

Proof.

Existence: in such an equilibrium, ND is off the equilibrium path. Thus, we can set $P(ND) = a$, in which case no firm strictly prefers to refrain from disclosure.

Uniqueness: By contradiction, suppose instead that there was an equilibrium in which the firm sometimes disclosed. Note it must be that $P(\tilde{x}) = \tilde{x}$. Thus, from equilibrium condition (b), a firm discloses if and only if $\tilde{x} > P(ND)$. Therefore,

$$P(ND) = \mathbb{E}(\tilde{x}|\tilde{x} \leq P(ND)).$$

But, note that:

$$\begin{aligned} \mathbb{E}(\tilde{x}|\tilde{x} \leq P(ND)) &= F(P(ND))^{-1} \left[\int_a^{P(ND)} x f_x(\cdot)\, dx \right] \\ &< F(P(ND))^{-1} \left[\int_a^{P(ND)} P(ND) f_x(\cdot)\, dx \right] \\ &= P(ND), \end{aligned}$$

a contradiction. □

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Costly Voluntary Disclosure: Setup

Early papers in voluntary disclosure theory demonstrate how to break the non-descriptive unravelling result.

- Jovanovic (1982) and Verrecchia (1983) add a cost $c \in (0, b - \mu)$ to disclosure. The original Verrecchia paper had risk aversion, though it won't be important for our analysis.
- The cost may represent a cost to produce/verify information or proprietary costs.
- Common criticism: why doesn't the cost depend on firm value? We will return to this in the context of product markets.

Definition

An equilibrium is a price function $P(\cdot)$ mapping the observed disclosure D to a price, and a disclosure strategy $D(\tilde{x})$ such that:
(a) $P(D) = \mathbb{E}[\tilde{x}|D(\tilde{x}) = D]$ on the equilibrium path;
(b) $D(\tilde{x}) = \tilde{x}$ if and only if $\tilde{x} \geq P(ND) + c$.

Costly Voluntary Disclosure: Existence

Theorem

In any equilibrium, the firm discloses if and only if $\tilde{x} \geq T$ where $T \in (a, b)$ solves:

$$T - c = \mathbb{E}\left[\tilde{x} | \tilde{x} \leq T\right]. \tag{1}$$

There exists at least one such equilibrium.

Proof.

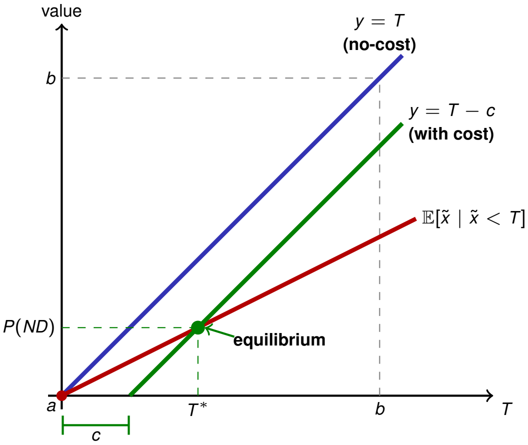
1. Note from condition (b) that any equilibrium must be characterized by a threshold $T = P(ND) + c$ such that a firm discloses if and only if $\tilde{x} > T$.
2. By condition (a), this implies that an equilibrium is characterized by $P(ND) = T - c = \mathbb{E}\left[\tilde{x} | \tilde{x} \leq T\right] \Leftrightarrow g(T) \equiv T - c - \mathbb{E}\left[\tilde{x} | \tilde{x} \leq T\right] = 0$.
3. To show that a solution in (a, b) exists, note that $g(T)$ is continuous and that $g(a) = -c < 0 < g(b) = b - c - \mu$. The result now follows from the intermediate-value theorem.

□

Note that if $c \geq b - \mu$, there is a unique equilibrium with no disclosure.

Visualizing Unravelling vs. Costly Disclosure

For concreteness take $\tilde{x} \sim U[a, b]$, so $\mathbb{E}[\tilde{x} \mid \tilde{x} < T] = (a + T)/2$. Plot both sides of the equilibrium condition against the threshold T:



- **No cost:** equilibrium needs $y = T$ (blue), but $\mathbb{E}[\tilde{x} \mid \tilde{x} < T]$ (red) lies strictly below for all interior $T \Rightarrow$ *unravelling*.
- **With cost c:** condition shifts down to $y = T - c$ (green), producing an interior intersection at $T^* \Rightarrow$ *equilibrium exists*.

Costly Voluntary Disclosure: Discussion

- Equation (1) need not have only one solution. Log-concavity of the distribution yields uniqueness because it ensures:

$$\frac{\partial}{\partial T}\mathbb{E}\left[\tilde{x}|\tilde{x} < T\right] < 1.$$

  Hence, $g'(T) = 1 - \frac{\partial}{\partial T}\mathbb{E}\left[\tilde{x}|\tilde{x} < T\right] > 0$. An example of a log-concave distribution is the normal. Bagnoli and Bergstrom (2005) is the classic reference on these results.

- Note that, given risk neutrality, disclosure never changes price in these models in expectation. Thus, incurred disclosure costs are a deadweight loss to the firm.
  - The firm would prefer to commit never to disclose.
  - However, from a societal point of view, whether disclosure is a deadweight loss depends upon the nature of the disclosure cost. If it is proprietary, disclosure could have a positive externality.
  - Bertomeu, Vaysman, and Xue (2021): optimal mandatory disclosure policy when firm can subsequently voluntarily disclose to minimize costs is “conservative.”

- From the firm’s perspective, a stronger result holds.

Costly Voluntary Disclosure: Discussion

Theorem

If there exist two equilibria, the one with a higher disclosure threshold is Pareto preferred to the one with the lower threshold.

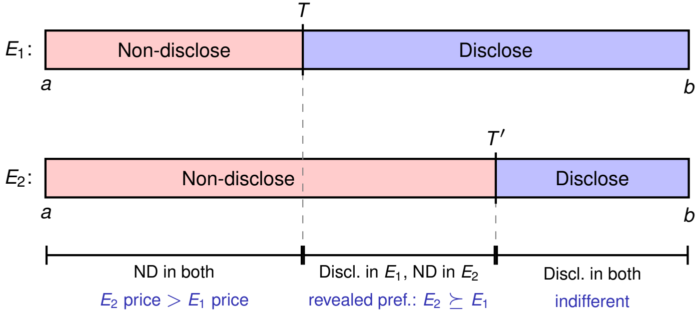
Proof.

Let the two equilibria be characterized by thresholds T and T', where $T < T'$; call them E_1 and E_2.

1. Firms with $\tilde{x} < T$ do not disclose in both E_1 and E_2, but receive a lower price under E_1.
2. Under E_2, firms with $x \in [T, T']$ could have disclosed but chose not to, and thus they must be better off by revealed preference.
3. Finally, firms $x > T'$ disclose under either equilibrium and are thus indifferent.

□

Costly Voluntary Disclosure: Visual Proof



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Dye (1985)/Jung and Kwon (1988)

- Dye (1985) is the next classic paper on how the unravelling result can be broken.

- Dye considers a model in which there is some probability p that the manager is not informed. The uninformed manager has no means to credibly convey this.
  - Could also be that the manager does not care about price, or is informed but cannot credibly convey their info.

- Intuition for why this breaks unravelling:
  - Interpret the model as one with many identical firms.
  - In equilibrium, suppose managers with bad news can refrain from disclosure.
  - The market then doesn't know if non-disclosure comes from an uninformed manager or a bad-news manager.
  - Since uninformed managers have average news, this allows the bad news managers to obtain an inflated price.

Dye (1985)/Jung and Kwon (1988)

Any equilibrium will again take a threshold form. The equilibrium is again defined by indifference of the threshold firm (why?):

$$T = \mathbb{E}\left[\tilde{x}|ND\right].$$

Let $\tilde{I} = 1$ (0) when the manager is informed (uninformed). Given this setup, note the price given non-disclosure can be calculated as follows:

$$\begin{aligned} P(ND) &= \mathbb{E}\left[\tilde{x}|ND\right] \\ &= \Pr\left(\tilde{I} = 0|ND\right)\mathbb{E}\left[\tilde{x}|\tilde{I} = 0, ND\right] + \Pr\left(\tilde{I} = 1|ND\right)\mathbb{E}\left[\tilde{x}|\tilde{I} = 1, ND\right] \\ &= \frac{p}{p + (1-p)F(T)}\mu + \frac{(1-p)F(T)}{p + (1-p)F(T)}\mathbb{E}\left[\tilde{x}|\tilde{x} < T\right] \\ &= \frac{p\mu + (1-p)\int_a^T tf(t)dt}{p + (1-p)F(T)}. \end{aligned}$$

Indifference of the threshold firm can be written as $P(T) - P(ND) = 0$, or

$$g(T) \equiv T - \frac{p\mu + (1-p)\int_a^T tf(t)dt}{p + (1-p)F(T)} = 0.$$

Dye (1985)/Jung and Kwon (1988)

Theorem

There exists a unique equilibrium characterized by disclosure threshold $T \in (a, \mu)$. Further, T is increasing in p.

Note: disclosure threshold is always below the prior mean μ. Why?

Proof.

Existence. This follows by applying the intermediate-value theorem:

$$g(a) = a - \frac{p\mu + (1-p)\int_a^a tf(t)dt}{p + (1-p)F(a)} = a - \mu < 0.$$

$$g(\mu) = \mu - \underbrace{\frac{p\mu + (1-p)F(\mu)\mathbb{E}[\tilde{x}|\tilde{x} < \mu]}{p + (1-p)F(\mu)}}_{\text{weighted average of } \mu \text{ and } \mathbb{E}[\tilde{x}|\tilde{x}<\mu] < \mu} > 0.$$

Dye (1985)/Jung and Kwon (1988)

Proof continued.

Uniqueness. Integrating by parts yields that:

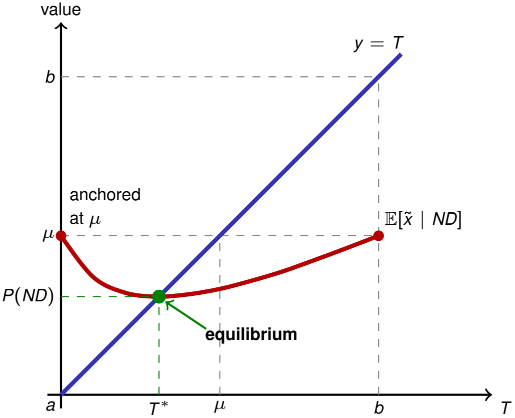
$$\begin{aligned} g(T) &= T - \frac{p\mu + (1-p)\int_a^T tf(t)dt}{p + (1-p)F(T)} \\ &= T - \frac{p\mu + (1-p)(TF(T) - \int_a^T F(t)dt)}{p + (1-p)F(T)} \\ &= \frac{p(T-\mu) + (1-p)\int_a^T F(t)dt}{p + (1-p)F(T)}. \end{aligned}$$

This has the sign of the numerator (since the denominator is positive). Moreover, the numerator increases in T.

Alternative proof: Suppose not, i.e., there exist two equilibria characterized by thresholds $T < T'$. Then, T' is Pareto-preferred to T (the proof is identical to the cost case). But the expected price must be $\mathbb{E}[\tilde{x}]$ from iterated expectations under both equilibria, a contradiction. □

Visualizing the Dye/Jung-Kwon Equilibrium

$\mathbb{E}[\tilde{x} \mid ND]$ is a weighted average of μ (if uninformed, weight p) and $\mathbb{E}[\tilde{x} \mid \tilde{x} < T]$ (if informed, weight $(1-p)F(T)$). The endpoints *anchor* the curve at μ:



- At $T = a$: only uninformed managers withhold $\Rightarrow \mathbb{E}[\tilde{x} \mid ND] = \mu$, which lies *above* the 45° line.
- At $T = \mu$: curve dips below μ (weighted avg. of μ and $\mathbb{E}[\tilde{x} \mid \tilde{x} < \mu] < \mu$) $\Rightarrow$ *below* the 45° line.
- Continuity $\Rightarrow$ unique crossing at some $T^* \in (a, \mu) \Rightarrow$ *interior equilibrium exists*.
- Equilibrium actually occurs at the minimum of $\mathbb{E}[\tilde{x} \mid ND]$ across all possible disclosure policies (Acharya, Demarzo, and Kremer 2011)

A Review (?): Stochastic Dominance

Definition

$\tilde{x}_1$ first-order (second-order) stochastically dominates $\tilde{x}_2$ if and only if, $\forall x$, $F_1(x) \leq F_2(x)$ $\left(\int_a^x F_1(t)dt \leq \int_a^x F_2(t)dt\right)$, with inequality strict on some interval.

- Often written $\tilde{x}_1 \succeq_{FSD} \tilde{x}_2$ ($\tilde{x}_1 \succeq_{SSD} \tilde{x}_2$).
- This is not a function of the random variable itself but of its distribution (these are examples of "stochastic orders;" there are many such orders).

Theorem

$\tilde{x}_1 \succeq_{FSD} \tilde{x}_2$ is equivalent to either of the following conditions:

1. $\mathbb{E}\left[u\left(\tilde{x}_1\right)\right] > \mathbb{E}\left[u\left(\tilde{x}_2\right)\right]$ *for any increasing function* $u\left(\cdot\right)$*;*
2. $\tilde{x}_2 =^d \tilde{x}_1 - \tilde{\delta}$ *for some non-negative, non-zero random variable* $\tilde{\delta}$*.*

A Review (?): Stochastic Dominance

Theorem

$\tilde{x}_1 \succeq_{SSD} \tilde{x}_2$ is equivalent to either of the following conditions:

1. $\mathbb{E}\left[u\left(\tilde{x}_1\right)\right] > \mathbb{E}\left[u\left(\tilde{x}_2\right)\right]$ *for any increasing concave function* $u\left(\cdot\right)$*;*
2. $\tilde{x}_2 =^d \tilde{x}_1 + \tilde{\varepsilon} - \tilde{\delta}$ *for some non-negative, non-zero random variable* $\tilde{\delta}$ *and some* $\tilde{\varepsilon}$ *with* $\mathbb{E}\left[\tilde{\varepsilon}|\tilde{x}_1 - \tilde{\delta}\right] = 0$*.*

Note:

- $=^d$ refers to equality in distribution.
- $\tilde{x}_1 \succeq_{FSD} \tilde{x}_2 \implies \tilde{x}_1 \succeq_{SSD} \tilde{x}_2$; Why?
- $\tilde{x}_1 \succeq_{FSD} \tilde{x}_2$ or $\tilde{x}_1 \succeq_{SSD} \tilde{x}_2 \implies \mathbb{E}\left(\tilde{x}_1\right) \geq \mathbb{E}\left(\tilde{x}_2\right)$. Why?

A Review (?): Stochastic Dominance

Proof idea in the FSD case.

To start, let's show that $F_1 < F_2 \Rightarrow \mathbb{E}\left[u(\tilde{x}_1)\right] > \mathbb{E}\left[u(\tilde{x}_2)\right]$, for any u with $u' > 0$. Integrating by parts gives:

$$\begin{aligned}\mathbb{E}\left[u(x)\right] = \int_a^b u(x)f(x)dx &= u(x)F(x)\Big|_a^b - \int_a^b u'(x)F(x)dx \\ &= u(b)F(b) - u(a)F(a) - \int_a^b u'(x)F(x)dx \\ &= u(b) - \int_a^b u'(x)F(x)dx.\end{aligned}$$

So, we get:

$$\begin{aligned}\mathbb{E}\left[u(\tilde{x}_1)\right] - \mathbb{E}\left[u(\tilde{x}_2)\right] &= \int_a^b u'(x)F_2(x)dx - \int_a^b u'(x)F_1(x)dx \\ &= \int_a^b u'(x)(F_2(x) - F_1(x))dx. \qquad (2)\end{aligned}$$

If $u' > 0$, we see that expected utility is higher under F_1 if $F_1 < F_2$.

How do we go the other way?

Suppose by contradiction that $F_2 < F_1$ on some interval (l, h).

Consider a utility function that has $u'(x) = H$ on (l, h) and $u'(x) = \varepsilon$ on $(a, b) \setminus (l, h)$, for H large and ϵ small. Applying (2), we get $\mathbb{E}\left[u(\tilde{x}_1)\right] < \mathbb{E}\left[u(\tilde{x}_2)\right]$, a contradiction.

Stochastic Dominance in Jung and Kwon

Theorem

Let T_1 and T_2 denote the disclosure thresholds under $\tilde{x}_1$ and $\tilde{x}_2$. If $\tilde{x}_1$ FSD or SSD dominates $\tilde{x}_2$, then $T_1 \geq T_2$.

Proof.

Since $\succeq_{FSD}$ implies $\succeq_{SSD}$, it is sufficient to show the result for SSD. Recall the equilibrium condition was:

$$\frac{p(T-\mu) + (1-p)\int_a^T F(t)dt}{p + (1-p)F(T)} = 0.$$

Getting rid of the denominator and simplifying, we have that the equilibria under the two distributions are defined by:

$$\begin{aligned}
\mu_1 - T_1 &= \frac{1-p}{p}\int_a^{T_1} F_1(t)dt; \\
\mu_2 - T_2 &= \frac{1-p}{p}\int_a^{T_2} F_2(t)dt.
\end{aligned}$$

Stochastic Dominance in Jung and Kwon

Proof continued.

Now, suppose by contradiction that $T_1 < T_2$. Then, by the definition of SSD,

$$\begin{aligned}
\mu_2 - T_2 &= \frac{1-p}{p}\int_a^{T_2} F_2(t)dt \\
&> \frac{1-p}{p}\int_a^{T_2} F_1(t)dt \\
&> \frac{1-p}{p}\int_a^{T_1} F_1(t)dt = \mu_1 - T_1.
\end{aligned}$$

Therefore, $\mu_1 - \mu_2 < T_1 - T_2 < 0$, contradicting the fact that $\tilde{x}_1 \succeq_{SSD} \tilde{x}_2 \implies \mathbb{E}[\tilde{x}_1] \geq \mathbb{E}[\tilde{x}_2]$. □

Dye (1985)/Jung and Kwon (1988)

Theorem

Suppose $\tilde{x} \sim N(\mu, \sigma^2)$. Then, the probability of disclosure is independent of μ and σ^2.

Follows more generally for location/scale parameterizations.

Intuition for μ: simply shifts around the prior mean without affecting inferences from disclosure.

- Important since empirical work sometimes argues that more profitable firms are more likely to disclose for instance, Li (2008) argues that less profitable firms as measured by lower contemporaneous ROA are less likely to disclose
- If it is common knowledge such firms are more profitable (for instance, profitability captured by ROA), this is not well supported by rational theory

Dye (1985)/Jung and Kwon (1988)

Theorem

Suppose $\tilde{x} \sim N(\mu, \sigma^2)$. Then, the probability of disclosure is independent of μ and σ^2.

Intuition for σ^2:

- Fixing the disclosure threshold $T < \mu$, a greater variance reduces the likelihood of disclosure since fewer types fall above the threshold
- However, the conditional expectation of firm value below a threshold goes down:

$$\frac{\partial}{\partial \sigma} \mathbb{E}[\tilde{x} | \tilde{x} < T] < 0.$$

  pushing down the non-disclosure price.
- This pushes down the disclosure threshold by an exactly offsetting amount.

Dye (1985)/Jung and Kwon (1988)

Proof.

Let $\Phi(\cdot)$ denote the CDF of a standard normal. The equilibrium condition is:

$$p\,(T-\mu) = -\,(1-p)\int_{-\infty}^{T}\Phi\left(\frac{t-\mu}{\sigma}\right)dt$$

$$\Leftrightarrow p\,(T-\mu) = -\,(1-p)\int_{-\infty}^{\frac{T-\mu}{\sigma}}\sigma\Phi\,(t)\,dt$$

$$\Leftrightarrow \frac{T-\mu}{\sigma} + \frac{1-p}{p}\int_{-\infty}^{\frac{T-\mu}{\sigma}}\Phi\,(t)\,dt = 0.$$

This implies $T = \sigma * T_{norm} + \mu$ where T_{norm} solves:

$$T_{norm} + \frac{1-p}{p}\int_{-\infty}^{T_{norm}}\Phi(t)dt = 0.$$

Now,

$$\Pr\,(Disc) = (1-p)\Pr\,(\tilde{x} > T) = (1-p)\Pr\left(\frac{\tilde{x}-\mu}{\sigma} > T_{norm}\right)$$
$$= (1-p)\left[1-\Phi\,(T_{norm})\right].$$

□

Other Work on Voluntary Disclosure

These models have been extended in many ways. Just a few:

- Einhorn and Ziv (2008). Dynamic Jung and Kwon setting in which the probabilities of being informed in each period are correlated, and there is a cost to disclosure. Creates autocorrelation in disclosure.
- Einhorn and Ziv (2012) "Biased voluntary disclosure." Incorporates the possibility of bias conditional on disclosure.
- Acharya, Demarzo, and Kremer (2011). Dynamic disclosure when there is an upcoming public release. There is an option value to wait.
- Marinovic and Varas (2016). Dynamic disclosure model where information changes over time. Considers litigation risk.
- Banerjee, Marinovic and Smith (2022). Dye and Verrecchia voluntary disclosure models in a Hellwig market.

Empirical Applications

In many empirical settings, managers clearly possess info (i.e., $p = 1$). Disclosure costs more likely to explain non-disclosure.

- Verrecchia and Weber (2006), Boone et al. (2016). Firms frequently redact information from contracts that they are required to present in their SEC filings.
- Berger and Hann (2007) study managers' tendency to withhold segment-level performance. Internal accounting systems let managers observe the breakdown of their performance into segment-level earnings $\Rightarrow$ they argue that agency and proprietary costs lead managers to withhold in this setting.
- Gow, Larcker, and Zakolyukina (2021) find that managers sometimes refuse to answer questions during conference calls that ask for monetary amounts, locations, and times, and attribute this to an unwillingness to reveal proprietary information.
- Glaeser (2018) & Saidi and Zaldokas (2021) study firms' decisions to patent technologies that they are known to possess, as Regulation S-K requires them to disclose the presence (but not specifics) of such technologies.

Empirical Applications

In other settings, disclosure costs may be minimal and it may be unclear to investors whether managers possess verifiable information

$\Rightarrow$ Dye framework may be appropriate.

- Proprietary costs are likely to be negligible for firms who enjoy secure monopoly power.
- E.g., pharma/bio-tech: firms invest in R&D, such as clinical trials, that if successful, provides them with monopoly power
- The outcome of such R&D often produces verifiable results with large implications for firm value that arrive at unknown times.

The common application of these frameworks to earnings guidance is less clear.

- How could a firm not have some worthwhile information about earnings?

Application to Earnings Guidance

- In reality, managers often disclose bad news.
- For example, Bertomeu, Marinovic, and Ma (2020, TAR) find, using structural estimation, that voluntary disclosure models cannot explain the frequency of bad news forecasts.
- The voluntary disclosure model assumes managers care exclusively about short-term price.
  - This is clearly not an important assumption when considering disclosures that do not affect cash flows/prices in the future.
- However, some disclosures might.
  - Managers may develop a reputation for being forthcoming by disclosing bad news. see Beyer and Dye (2012, RAST) for a model of this, which builds upon the economics literature on reputation
  - Managers may be concerned about litigation if they do not disclose news on a timely basis.
  - Disclosing bad news might impact long-term cash flows via the product market.

Exercise: Three State Voluntary Disclosure

Suppose that the manager observes $\tilde{x} \in \{\bar{x} - \delta, \bar{x}, \bar{x} + \delta\}$, where:

$$\Pr(\tilde{x} = \bar{x} - \delta) = \Pr(\tilde{x} = \bar{x} + \delta) = p/2;$$
$$\Pr(\tilde{x} = \bar{x}) = 1 - p.$$

The manager observes $\tilde{x}$ with probability $q \in (0, 1)$. Solve for the disclosure equilibrium.

First step: notice that the same logic from the previous, continuous version of the model that $T < \mathbb{E}[\tilde{x}] = \bar{x}$ also applies here.

1. The manager wants to disclose upon observing $\tilde{x} = \bar{x} + \delta$.
2. Hence, in any equilibrium, $T < \bar{x} + \delta$, and so $\mathbb{E}[\tilde{x} | \tilde{x} < T] < \bar{x}$.
3. This means that $\mathbb{E}[\tilde{x} | ND]$, being a weighted average of $\mathbb{E}[\tilde{x} | \tilde{x} < T] < \bar{x}$ and $\bar{x}$, is also less than $\bar{x}$.
4. Thus, when the manager observes $\tilde{x} = \bar{x}$ she also wants to disclose.

Exercise: Three State Voluntary Disclosure

Suppose that the manager observes $\tilde{x} \in \{\bar{x} - \delta, \bar{x}, \bar{x} + \delta\}$, where:

$$\Pr(\tilde{x} = \bar{x} - \delta) = \Pr(\tilde{x} = \bar{x} + \delta) = p/2;$$
$$\Pr(\tilde{x} = \bar{x}) = 1 - p.$$

The manager observes $\tilde{x}$ with probability $q \in (0, 1)$. Solve for the disclosure equilibrium.

Notice this leaves two possibilities: when the manager is informed, either there is full disclosure, or non-disclosure only when $\tilde{x} = \bar{x} - \delta$.

Next step: Show full disclosure cannot be an equilibrium.

1. Suppose we were in an equilibrium with full disclosure.
2. In this case, investors would know non-disclosure only arises when the manager is uninformed, so that $\mathbb{E}[\tilde{x}|ND] = \bar{x}$.
3. But, this means the manager prefers to deviate to not disclosing when she observes $\tilde{x} = \bar{x} - \delta$.

Exercise: Three State Voluntary Disclosure

Suppose that the manager observes $\tilde{x} \in \{\bar{x} - \delta, \bar{x}, \bar{x} + \delta\}$, where:

$$\Pr(\tilde{x} = \bar{x} - \delta) = \Pr(\tilde{x} = \bar{x} + \delta) = p/2;$$
$$\Pr(\tilde{x} = \bar{x}) = 1 - p.$$

The manager observes $\tilde{x}$ with probability $q \in (0, 1)$. Solve for the disclosure equilibrium.

Final step: Verify that disclosure given $\tilde{x} \in \{\bar{x}, \bar{x} + \delta\}$ is in fact an equilibrium.

1. The non-disclosure price is:

$$P_{ND} = \frac{q * (p/2) * (\bar{x} - \delta) + (1 - q) * \bar{x}}{q * (p/2) + (1 - q)}.$$

2. Observe that this is a weighted average of $\bar{x} - \delta$ and $\bar{x}$. So, $P_{ND} \in (\bar{x} - \delta, \bar{x})$.
3. Hence, it is indeed the case that the manager wants to disclose when $\tilde{x} \in \{\bar{x}, \bar{x} + \delta\}$ and not disclose when $\tilde{x} = \bar{x} - \delta$.

Agenda

- Unravelling
- Verrecchia (1983)
  Costly disclosure
- Dye (1985)/Jung and Kwon (1988)
  Probabilistic information endowment
- Product Markets
  Endogenizing proprietary costs

Disclosure and Product Markets

Many papers consider how firms disclose when their disclosure influences the production of other firms.

These papers find very different results depending upon the assumptions.

Possible assumptions:

1. **Who are the competitors.** Entrants or existing competitors?
2. **Nature of competition.** Bertrand or Cournot?
3. **Nature of private information.** Cost or demand?
4. **Nature of communication game.** Dye/J&K? Verrecchia? Ex-ante commitment? Information design?

Disclosure and Product Markets

Perhaps the most important condition: Is good news for the firm good or bad news for a competitor/entrant?

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1. Suppose good news for the firm is *neutral or bad* news for competitors, leading them to produce less ("idiosyncratic" info)
   - Then, both financial and product market incentives push towards disclosing good news, and we get unravelling (under voluntary disclosure)
   - May reflect good news on a firm's marginal costs (i.e., proprietary innovations, contracts with supply chain partners)
   - May reflect, e.g., a disclosure that reveals the firm has made significant progress on technological development
2. Suppose good news for the firm is *good* news for competitors, leading them to produce more ("systematic" info)
   - Then, we get a proprietary cost to disclosing good news, and we tend to have non-disclosure in equilibrium
   - May reflect information on demand for a common product (e.g., past revenues)

Entry Games

One natural case is where the firm's primary goal is to deter new entrants (may apply especially to monopolies).

- Disclosing systematic positive news is likely to draw new entrants.

- Darrough and Stoughton (1990): one potential entrant, profits lower for the incumbent if they enter. Binary firm values.
  - Unravelling when the prior of the market is optimistic or the entry cost is relatively low. In this case, the entrant will probably enter either way, so financial market incentives are stronger.
  - When the prior is pessimistic or the entry cost is high, however, non- and partial-disclosure equilibria arise. In this case, the entrant's decision hinges on the firm's disclosure choice.

- Wagenhofer (1990): considers a more general discrete action that some other party can take that harms the disclosing firm and that is more likely given more positive info finds equilibria where the firm discloses only moderate values

Competition among Existing Producers: Darrough (1993)

- Analyzes eight settings:
  - Ex-ante and ex-post disclosure equilibria
  - Efficient disclosure choices across Bertrand and Cournot
  - Cost or demand information
- The paper has many results, effectively, one for each of these settings.
- We will survey the assumptions and go through some of these results.

Darrough (1993): Model

- Assume 2 risk-neutral firms compete.
- Each firm has a technology that produces differentiated goods at a fixed marginal cost c and faces an inverse demand curve:

$$P_i = a + \Delta\tilde{a} - Q_i - t\sum_{j \neq i} Q_j,$$

where:

  - P_i is the price paid for the goods of firm i,
  - $a + \Delta\tilde{a}$ is a common-market demand intercept.
  - Q_j represents each firm's output, and $t \in [0, 1]$ represents the degree of product differentiation, ranging from 0 when the goods are independent to 1 when the goods are perfect substitutes.

Darrough (1993): Model

- Firms observe signals:

$$\begin{aligned} x_i &= \Delta a_i + e_i \text{ with } e_i \sim N(0, m) \\ \Delta a &= \frac{1}{2}(\Delta a_1 + \Delta a_2), \end{aligned}$$

  where $\Delta a_1, \Delta a_2 \sim N(0, \sigma)$ are independent. Thus they observe independent noisy signals.

- Firms disclose signals $\hat{x}_i = x_i + f_i$ where $f_i \sim N(0, s_i)$.

- Marginal costs $\Delta c_i \sim N(0, \sigma_c)$.

- Firms observe signals:

$$z_i = \Delta c_i + \varepsilon_i \text{ with } \varepsilon_i \sim N(0, n)$$

  and report $\hat{z}_i = z_i + g_i$ where $g_i \sim N(0, v_i)$.

Darrough (1993): Overview

Part (i): Commit or not?

- The paper first analyzes a setting where firms can choose whether to commit to disclosing their signal ex-ante.
- They look for Nash equilibria where firms both decide on the optimal precision of their public signal s_i (or later, v_i).
- The firms' only goal is to maximize profits.
- They compare this to what the agents would do if they could coordinate their disclosure decisions.

Part (ii): Voluntary disclosure

- The paper then studies the voluntary disclosure case, where firms cannot commit.

We will start with the results from Part (i).

Cournot Demand Uncertainty

1. Firms simultaneously first decide on disclosure.
2. Firms release disclosures.
3. Firms choose quantities to produce.

Solve via backwards induction. Production stage is the standard quadratic optimization problem you are familiar with from micro. Yields:

$$\begin{aligned} Q_i &= \frac{1}{2+t}\left[a + \frac{1}{2}E\left(\Delta a_j | \hat{x}_j\right) - \frac{t}{4}E\left(\Delta a_i | \hat{x}_i\right) + \frac{2+t}{4}E\left(\Delta a_i | x_i\right)\right] \\ &= \frac{1}{2+t}\left[a + \frac{\sigma}{2\left(m+\sigma+s_j\right)}\hat{x}_j - \frac{t\sigma}{4\left(m+\sigma+s_i\right)}\hat{x}_i + \frac{\sigma\left(2+t\right)}{4\left(m+\sigma\right)}x_i\right]. \end{aligned}$$

Moreover, expected profits equal $E\left[Q_i^2\right]$. Thus, they increase in variability in Q_i.
Intuition: The firm varies Q_i to take advantage of known demand shocks. Q_i more variable means the firm is exploiting more variation in demand.

Cournot Demand Uncertainty

$$Q_i = \frac{1}{2+t}\left[a + \frac{\sigma}{2(m+\sigma+s_j)}\hat{x}_j - \frac{t\sigma}{4(m+\sigma+s_i)}\hat{x}_i + \frac{\sigma(2+t)}{4(m+\sigma)}x_i\right].$$

- Fixing the other firm's disclosure choice, disclosing reduces variability in Q_i.
  - See highlighted term. Disclosing high x_i leads the firm's competitor to produce more. But, this leads the firm to pull back, reducing variation in Q_i.
  - The unique *Nash* equilibrium is not to commit to disclose.
- But, knowing about my competitor's production leads to more variability in my own production, because it leads to more variation in $\hat{x}_j$, and thus greater profit; see highlighted term.
- If t is sufficiently small, both firms want to commit to arrange to share information.
  - The benefit to obtaining information from the competitor is greater than the cost to disclosing information to the competitor.
  - Spillover to other firm is small; information gain is large.

Bertrand Demand Uncertainty

1. Firms simultaneously first decide on disclosure.
2. Firms release disclosures.
3. Firms choose **prices.** Note the economic difference between Cournot and Bertrand is flexibility in production. Cournot (Bertrand) better applies to cases where production is inflexible (flexible).

- Again, solve via backwards induction. Some algebra yields:

$$P_i = \frac{1-t}{2-t}\left[a + \frac{1}{2}\frac{\sigma}{m+\sigma+s_j}\hat{x}_j + \frac{t\sigma}{4\left(m+\sigma+s_i\right)}\hat{x}_i + \frac{\sigma\left(2-t\right)}{4\left(m+\sigma\right)}x_i\right].$$

  In Bertrand, decisions are strategic complements. Disclosure that raises the competitors' production also leads me to produce more $\Rightarrow$ ex ante, disclosure leads to more variability in Q_i and thus profits.

- Basic idea is that competition is more intense given flexibility in production, and companies really benefit from being able to coordinate their decision making.

- Nash disclosure policies are perfect ($s_i = 0$).

Demand Known, Costs Unknown

Firms' decisions in the production stage are unchanged, but they vary differently with the firms' disclosures. The results reverse:

- **Cournot:** Nash equilibrium is full disclosure.
- **Bertrand:** Nash equilibrium is no disclosure.

Key difference: demands are common values, costs are private values.

- **Cournot:** Suppose the firm discloses a lower marginal cost. Then, the competitor knows the firm will produce more and react by producing less. Thus, disclosure raises the variation in Q_i.[1]
- **Bertrand:** If the firm discloses a lower marginal cost, they will lower price. This will lead the competitor to also lower its price, reducing the firm's quantity sold. Thus, disclosure lowers the variation in Q_i.

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[1] Alternative intuition: sps. costs are either very low or very high. By disclosing, the firm gets to behave as a monopolist when it has a low cost and the other firm has a high cost. It is better to be a monopolist half the time and not in the market half the time than to be in a duopoly.

Ex-Post (Voluntary Disclosure) Setting

- Suppose firm *i* receives a signal with probability γ.

- Cournot/cost and Bertrand/demand settings are straightforward.
  - In both cases, the firm's benefit from disclosure is monotonic. Thus, standard voluntary disclosure arguments apply.
  - The paper claims that one gets unravelling, but this seems to only be true when $\gamma = 1$. Otherwise, we should get a Dye-type equilibrium.

- Cournot/demand and Bertrand/cost settings are more subtle.

Ex-Post Cournot/Demand

Let $X_i^D(\gamma)$ denote the disclosure region for a given γ. $\gamma = 1$ case:

Proposition 5: In Cournot competition with demand uncertainty, the Bayesian-Nash equilibrium strategy of firm *i* in the *ex post* setting in which firms are expected to possess private information is characterized by:

$$X_i^D(1) = \left(\frac{t\bar{x}_i - 8a}{t+4}, \bar{x}_i\right),$$

where $\bar{x} = E[x_i | x_i \in X_i^{ND}(1)]$ and $0 < \bar{x}_i < 8a/t$.[24]

- This case is studied in detail in Clinch and Verrecchia (1997). Even when the manager is known to be informed, we do not get unravelling.
- Extreme values are withheld. The disclosing firm benefits from this because the competitor makes the "wrong" decision: too high production for low demand, too low production for high demand.
- This argument only works when production can be negative. Essentially, the firm only refrains from disclosing low demand because she can prompt the competitor to overproduce, leading to very low prices. To benefit from this, the firm has to be able to sell negative quantities.
- One interpretation: buying/storing product purchased on the market.
- When γ shrinks, the disclosure region changes, but still looks qualitatively similar.

Ex-Post Bertrand/Cost

The ex-post/Bertrand setting is similar: extreme values are withheld.

Proposition 7: In Bertrand competition with cost uncertainty, the Bayesian-Nash equilibrium strategy of firm i in the *ex post* setting when firms receive private information for certain or with arbitrarily low probability are characterized by:

$$Z_i^D(1)=\begin{cases}\left[\bar{z}_i,\dfrac{t^2\bar{z}_i+4a(2+t)(1-t)}{8-3t^2}\right], & \forall t<1,\\ \emptyset & t=1,\end{cases}$$

and

$$\lim_{\gamma\to 0} Z_i^D(\gamma)=\begin{cases}\left[0,\dfrac{4a(2+t)(1-t)}{8-3t^2}\right], & \forall t<1,\\ \emptyset & t=1,\end{cases}$$

Discussion

Some takeaways:

- We need to be cautious about modeling proprietary costs as a constant *c*.
  - If we formally model disclosure's impact on competitors' production, the equilibrium may not be a "single threshold" equilibrium.
- Unfortunately, the predictions that come from actually modeling proprietary costs are very sensitive to assumptions, which makes these models difficult to empirically test. but, this challenge means there is opportunity for more research
- Some forms of disclosure costs may be better captured by a constant *c*.
  - E.g., verification costs. For instance, a firm may have a good idea of its carbon emissions but need to hire an auditor to credibly disclose them to the market.
  - Disclosing certain information may benefit competitors in a fairly simple way, not by telling them how to change production. E.g., revealing details that help the competitor in their strategy, or that are relevant for competitors' technological development.